Section 1.2

Mathematical Model

A mathematical representation of a particular situation is called a mathematical model.

Cost, Revenue, and Profit Functions

A cost function specifies the cost C as a function of the number of items x. Thus, $C(x)$ is the cost of x items. A cost function of the form

$$C(x) = mx + b$$

is called a linear cost function. The quantity mx is called the **variable cost** and the intercept b is called the **fixed cost**. The slope m, the marginal cost, measure the incremental cost per item.

The **revenue** resulting from one or more business transactions is the total payment received, sometimes called the gross proceeds. If $R(x)$ is the revenue from selling x items at a price of m each, then R is the linear function $R(x) = mx$ and the selling price m can also be called the **marginal revenue**.

The **profit** is what remains of the revenue when costs are subtracted. If the profit depends linearly on the number of items, the slope m is called the **marginal profit**. Profit, revenue, and cost are related by the following formula.

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$P = R - C$$

To **break even** means to make neither a profit nor a loss. Thus, break even occurs when $P = 0$, or

$$R = C.$$

The **break-even point** is the number of items x at which the break even occurs.

Demand, Supply, and Equilibrium Price

A **demand equation** or **demand function** expresses demand q (the number of items demanded) as a function of the unit price p (the price per item). A **supply equation** or **supply function** expresses supply q (the number of items a supplier is willing to make available) as a function of the unit price p (the price per item). It is usually the case that demand decreases and supply increases as the unit price increases.

Demand and supply are said to be in **equilibrium** when demand equals supply. The corresponding values of p and q are called the **equilibrium price** and **equilibrium demand**.

Compound Interest

If an amount (**present value**) P is invested for t years at an annual rate of r, and if the interest is compounded (reinvested) m times per year, then the **future value** A is

$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt}.$$

Problem 1. The cost of renting tuxes for the Choral Society's formal is \$20 down, plus \$88 per tux. Express the cost C as a function of x, the number of tuxedos rented. Use your function to answer the following questions.

a) What is the cost of renting 2 tuxes?

b) What is the cost of the 2^{nd} tux?

c) What is the cost of the 4098^{th} tux?

d) What is the variable cost? What is the marginal cost?

Problem 2. Your college newspaper, *The Collegiate Investigator*, has fixed production costs of \$70 per edition and marginal printing and distribution costs of 40¢ per copy. *The Collegiate Investigator* sells for 50¢ per copy.

a) Write down the associated cost, revenue, and profit functions.

b) What profit (or loss) results from the sale of 500 copies of *The Collegiate Investigator*?

c) How many copies should be sold in order to break even?

Problem 3. The demand for Sigma Mu Fraternity plastic brownie dishes is

$$q(p) = 361{,}201 - (p + 1)^2$$

where q represents the number of brownie dishes Sigma Mu can sell each month at a price of p¢. Use this function to determine:

a) The number of brownie dishes Sigma Mu can sell each month if the price is set at 50¢.

b) The number of brownie dishes they can unload each month if they give them away.

c) The lowest price at which Sigma Mu will be unable to sell any dishes.

Problem 4. The total weekly revenue earned at Royal Ruby Retailers is given by

$$R(p) = -\frac{4}{3}p^2 + 80p$$

where p is the price (in dollars) RRR charges per ruby. Use this function to determine:

a) The weekly revenue, to the nearest dollar, when the price is set at $20/ruby.

b) The weekly revenue, to the nearest dollar, when the price is set at $200/ruby. (Interpret your result.)

c) The price RRR should charge in order to obtain a weekly revenue of $1,200.

Problem 5. Worldwide quarterly sales of Nokia cell phones was approximately $q = -p + 156$ million phones when the wholesale price was $\$p$.

a) If Nokia was prepared to supply $q = 4p - 394$ million phones per quarter at a wholesale price of $\$p$, what would be the equilibrium price?

b) The actual wholesale price was $105 in the fourth quarter of 2004. Estimate the projected shortage or surplus at that price.

Problem 6. You invest $10,000 in Rapid Growth Funds, which appreciate by 2% per year, with yields reinvested quarterly. By how much will you investment have grown after 5 years?

Problem 7. Calculate the future value of an investment of $10,000 at 1.5% per year, compounded weekly (52 times/year), after 5 years.

Problem 8. A fossil originally contained 104 grams of carbon 14. Estimate the amount of carbon 14 left in the sample after 20,000 years. Use the formula $C(t) = A(0.999879)^t$, where A is the original amount of carbon 14.

Homework for this section: Read section 1.2. Watch any videos (marked with in the e-book)

Also, do the tutorials (marked with in the e-book). Come to class with at least two questions related to what you read/watched. Do the following problems in preparation for the quiz: #9, 11, 13, 17, 20, 31, 37, 40, 55