Section 2.3

Sinking Fund

A sinking fund is an account earning compound interest into which you make periodic deposits. Suppose that the account has an annual interest rate of r compounded m times per year, so that $i = r/m$ is the interest rate per compounding period. If you make a payment of PMT at the end of each period, then the future value after t years, or $n = mt$ periods, will be

$$FV = PMT\frac{(1+i)^n - 1}{i}.$$

Payment Formula for a Sinking Fund

Suppose that an account has an annual rate of r compounded m times per year, so that $i = r/m$ is the interest rate per compounding period. If you want to accumulate a total of FV in the account after t years, or $n = mt$ periods, by making payments of PMT at the end of each period, then each payment must be

$$PMT = FV\frac{i}{(1+i)^n - 1}.$$

Present Value of an Annuity

An **annuity** is an account earning compound interest from which periodic withdrawals are made. Suppose that the account has an annual rate of r compounded m times per year, so that $i = r/m$ is the interest rate per compounding period. Suppose also that the account starts with a balance of PV. If you receive a payment of PMT at the end of each compounding period, and the account is down to \$0 after t years, or $n = mt$ periods, then

$$PV = PMT\frac{1 - (1+i)^{-n}}{i}.$$

Payment Formula for an Ordinary Annuity

Suppose that an account has an annual rate of r compounded m times per year, so that $i = r/m$ is the interest rate per compounding period. Suppose also that the account starts with a balance of PV. If you want to receive a payment of PMT at the end of each compounding period, and the account is down to \$0 after t years, or $n = mt$ periods, then

$$PMT = PV\frac{i}{1 - (1+i)^{-n}}.$$

Problem 1. Suppose you deposit $900 per month into an account that pays 4.8% interest, compounded monthly. How much money will you have after 9 months?

Problem 2. You have a retirement account with $2000 in it. The account earns 6.2% interest, compounded monthly, and you deposit $50 every month for the next 20 years. How much will be in the account at the end of those 20 years?

Problem 3. You want to set up an education account for your child and would like to have $75,000 after 15 years. You find an account that pays 5.6% interest, compounded semiannually, and you would like to deposit money in the account every six months. How large must each deposit be in order to reach your goal?

Problem 3. Tom has just won the lottery and decides to take the 20 year annuity option. The lottery commission invests his winnings in an account that pays 4.8% interest, compounded annually. Each year for those 20 years, Tom receives a check from the lottery commission for $250,000. What is the present value of Tom's winnings? (Notice that this would be the amount that Tom would get if he chose the lump-sum option). What is the total amount of money that Tom gets over the 20 year period?

Problem 4. Beth has just received an inheritance of $400,000 and would like to be able to make monthly withdrawals over the next 15 years. She decides on an annuity that pays 6.7%, compounded monthly. How much will her monthly payments be in order to draw the account down to zero at the end of 15 years?

Problem 5. Ozzy is working in a tire factory that offers a pension in the form of an annuity that pays 5% annual interest, compounded monthly. He wants to work for 30 years and then have a retirement income of $4000 per month for 25 years. How much do he and his employer together have to deposit per month into the pension fund to accomplish this?

Problem 6. Chris and Katie are buying a house and have taken out a 30 year, $200,000 mortgage at 6.8% interest, compounded monthly. What will their monthly payments be? How much money will they have actually paid at the end of 30 years? How much interest will they have paid?

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #5, 9, 15, 21, 25, 43