A bag contains three red marbles, two green ones, one fluorescent pink one, two yellow ones, and two orange ones. Suzan grabs four at random. Find the probabilities of the indicated events.

a) She gets all the red ones, given that she does not get the fluorescent pink one.

Solution: Let $A$ be the event that she gets all the red ones, and let $B$ be the event that she does not get the fluorescent pink one. We need to find $P(A | B) = \frac{P(A \cap B)}{P(B)}$. $P(A \cap B)$ is the probability that she gets all the red ones AND she does not get the pink one. To find this, we need $n(S)$ and $n(A \cap B)$. $n(S)$ is the total number of ways that you can choose 4 marbles from 10, so $n(S) = C(10, 4) = 210$. To find $n(A \cap B)$ we need the total number of ways that she can choose 3 red marbles from 3, then choose the fourth one from 6 (the last marble can’t be red and it can’t be fluorescent pink, and there are 6 non-red/non-fluorescent pink marbles). This is $n(E) = C(3, 3) \times C(6, 1) = 1 \times 6 = 6$. Thus, $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{6}{210} = \frac{1}{35}$.

To find $P(B)$, we need $n(B)$ ($n(S)$ is the same). $n(B)$ is the number of ways that she can choose 4 marbles from 9 (the only restriction is that she does not get the one fluorescent pink marble, so that leaves 9 marbles to choose from). So, $n(B) = C(9, 4) = 126$ and $P(B) = \frac{n(B)}{n(S)} = \frac{126}{210} = \frac{3}{5}$. 
Finally, we get
\[
P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/35}{\frac{3}{5}} = \frac{1}{35} \times \frac{5}{3} = \frac{1}{21}
\]

b) She gets one of each color other than fluorescent pink, given that she gets the fluorescent pink one.

Solution: If she gets the fluorescent pink marble, then there is only room for 3 more colors of marble (since she is grabbing 4 marbles). Since there are 4 other colors of marble, it is not possible for her to get one of each of the other colors AND get the fluorescent pink one. Thus, the probability of this event is 0 (it is an impossible event). Mathematically, this can be shown as follows: Let \( A \) be the event that she gets one of each color other than fluorescent pink, and let \( B \) be the event that she gets the fluorescent pink one. We need to find \( P(A \mid B) = \frac{P(A \cap B)}{P(B)} \). To find \( P(A \cap B) \), we need \( n(A \cap B) \), the total number of ways that she can get one marble of each color other than fluorescent AND get the fluorescent pink marble. There are no combinations of 4 marbles that satisfy these two events simultaneously (meaning \( A \cap B = \emptyset \), or in words \( A \) and \( B \) are disjoint). Thus, \( P(A \cap B) = 0 \), which means that
\[
P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0.
\]
c) She gets at least two red ones, given that she gets at least one green one.

Solution: Let $A$ be the event that she gets at least two red ones, and let $B$ be the event that she gets at least one green one. We need to find $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. $P(A \cap B)$ is the probability that she gets at least two red ones AND she gets at least one green one. To find this, we need $n(A \cap B)$ ($n(S)$ is the same as it was in the previous two parts). To find $n(A \cap B)$ we need to find the total number of ways that she can get at least two red ones and at least one green one. We have to consider a few cases here: Case 1 is the case where she gets exactly two red and exactly one green. There are $C(3, 2) \cdot C(2, 1) \cdot C(5, 1) = 30$ ways to do this (choose 2 of the 3 red, 1 of the 2 green, and choose the last marble from the five marbles that aren’t red or green). Case 2 is the case where she gets exactly two red and exactly two green. There are $C(3, 2) \cdot C(2, 2) = 3$ ways to do this (choose 2 red from 3, then choose 2 green from 2). Case 3 is the case where she gets exactly three red and exactly one green. There are $C(3, 3) \cdot C(2, 1) = 2$ ways that this can happen. Case 4 would be the case where she gets three red and two green, but this is impossible since she is only grabbing four marbles. Adding up cases 1 through 3 gives

$$n(A \cap B) = 30 + 3 + 2 = 35.$$ 

Thus, $P(A \cap B) = \frac{35}{210} = \frac{1}{6}$. 
Next, we need to find \( P(B) \), the probability that she gets at least one green. There are two cases to consider: case 1 is she gets exactly one green. There are \( C(2, 1) \cdot C(8, 3) = 112 \) ways to do this (choose 1 green from 2, then choose the other 3 marbles from the 8 non-green ones). Case 2 is where she gets exactly two green ones. There are \( C(2, 2) \cdot C(8, 2) = 28 \) ways to do this (choose 2 green from 2, then choose the last 2 from the 8 non-green marbles). Thus, \( n(B) = 112 + 28 = 140 \) and \( P(B) = \frac{140}{210} = \frac{2}{3} \). Finally,

\[
P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{2/3} = \frac{1}{6} \times \frac{3}{2} = \frac{1}{4}.
\]