## Supply and Demand Examples

1) Sales figures show that your company sold 1960 pen sets each week when they were priced at $\$ 1 /$ pen set, and 1800 pen sets each week when they were priced at $\$ 5 /$ pen set. What is the linear demand function for your pen sets?

Solution: Recall that a linear demand function has the form $q=m p+b$. We need to find $m$ and $b$. We are given two values for $p$ and two values for $q$. This gives two points in the form $(p, q)$, where $\left(p_{1}, q_{1}\right)=(1,1960)$ and $\left(p_{2}, q_{2}\right)=(5,1800)$. We can find the slope $m$ using the formula

$$
m=\frac{q_{2}-q_{1}}{p_{2}-p_{1}}=\frac{1800-1960}{5-1}=-40
$$

Now we have $q=-40 p+b$. We can use this to find $b$ (the $q$-intercept), by choosing one of the given points, plugging in the values for $p$ and $q$, and solving for $b$ :

$$
1960=-40(1)+b
$$

Solving gives $b=2000$. Thus, the linear demand equation is

$$
q=-40 p+2000
$$

Note that the slope $m=-40$ can be written as $m=\frac{-40}{1}$, which is the change in $q$ over the change in $p$. This can be interpreted to mean that for every $\$ 1$ increase in the price of a pen set, we can expect that sales will decrease by 40.
2) The following table shows worldwide sales of Nokia cell phones and their average wholesale prices in 2004.

| Quarter | Second | Fourth |
| ---: | :---: | :---: |
| Wholesale Price (\$) | 111 | 105 |
| Sales (millions) | 45.4 | 51.4 |

a) Use the data to obtain a linear demand function for (Nokia) cell phones, and use your demand equation to predict sales if Nokia lowered the price further to $\$ 103$.
b) Fill in the blanks: For every $\qquad$ increase in price, sales of cell phones decrease by
$\qquad$ units.

Solution for (a): Here the quantity $q$ is the number of sales. We want to find a linear demand function, which has the form $q=m p+b$. We can find $m$ by finding the slope of the line between the points $\left(p_{1}, q_{1}\right)=(111,45.4)$ and $\left(p_{2}, q_{2}\right)=(105,51.4)$ :

$$
m=\frac{51.4-45.4}{105-111}=-1
$$

This means that $q=-p+b$. We can now use one of the points to find $b$ :

$$
45.4=-111+b
$$

This gives $b=156.4$, so the linear demand equation is

$$
q=-p+156.4
$$

We can now use this equation to predict what the sales would be if the price were lowered to $\$ 103$, by plugging in $p=103$ into the demand equation:

$$
q=-103+156.4=53.4
$$

Taking units into account from the table, this means that if the price were lowered to $\$ 103$, we could expect sales of about 53.4 million phones. This is of course consistent with the Law of Demand which states that lower prices lead to higher demand.

Solution for (b): The slope can be written as $m=\frac{-1}{1}$, which is the change in $q$ over the change is $p$. Considering that sales are given in millions on the table, this can be interpreted to mean that for each increase of $\$ 1$ in the price of a phone, we can expect sales to decrease by 1 million phones.
3) You can sell 90 pet chias per week if they are marked at $\$ 1$ each, but only 30 each week if they are marked at $\$ 2$ per chia. Your chia supplier is prepared to sell you 20 chias each week if they are marked at \$1/chia, and 100 each week if they are marked at $\$ 2$ per chia.
a) Write down the associated linear demand and supply functions.
b) At what price should the chias be marked so that there is neither a surplus nor a shortage of chias?

Solution for (a): We need to find both the supply and demand equations. Both have the same form, $q=m p+b$, but the slope on the demand equation is negative while the slope on the supply equation is positive. To find the demand equation, we use the two price/quantity pairs $(1,90)$ and $(2,30)$. We can find $m$ using the usual equation:

$$
m=\frac{q_{2}-q_{1}}{p_{2}-p_{1}}=\frac{30-90}{2-1}=-60
$$

This means $q=-60 p+b$. We can find $b$ by choosing one of the two price/quantity pairs, plugging them into the demand equation, and solving for $b$ :

$$
90=-60(1)+b
$$

Solving gives $b=150$. Thus the demand equation is $q=-60 p+150$.

We can find the supply equation in the same way - first finding the slope and then the $q$-intercept. Here we use the price quantity pairs $(1,20)$ and $(2,100)$. We find $m$ in the usual way:

$$
m=\frac{q_{2}-q_{1}}{p_{2}-p_{1}}=\frac{100-20}{2-1}=80
$$

This means $q=80 p+b$. We can find $b$ by choosing one of the two price/quantity pairs for supply, plugging them into the supply equation, and solving for $b$ :

$$
20=80(1)+b
$$

Solving gives $b=-60$. Thus the supply equation is $q=80 p-60$.
Solution for (b): To find the equilibrium price, we set supply equal to demand and then solve for $p$ :

$$
-60 p+150=80 p-60
$$

Solving for $p$ gives $p=1.5$. Thus the equilibrium price is $\$ 1.50$ per chia. If they are sold for this price, there will be neither a surplus nor a shortage.

