Section 1.5

**Vertical Line Test for Functions:** A set of points in a coordinate plane is the graph of \( y \) as a function of \( x \) if and only if no vertical line intersects the graph at more than one point.

**Zeros of a Function:** The zeros of a function \( f \) of \( x \) are the \( x \)-values for which \( f(x) = 0 \).

**Increasing and Decreasing Functions**

a) A function \( f \) is **increasing** on an interval if, for any \( x_1 \) and \( x_2 \) in the interval, \( x_1 < x_2 \) implies \( f(x_1) < f(x_2) \).

b) A function \( f \) is **decreasing** on an interval if, for any \( x_1 \) and \( x_2 \) in the interval, \( x_1 < x_2 \) implies \( f(x_1) > f(x_2) \).

c) A function \( f \) is constant on an interval if, for any \( x_1 \) and \( x_2 \) in the interval, \( f(x_1) = f(x_2) \).

**Relative Maximum and Relative Minimum**

A function value \( f(a) \) is called the relative minimum of \( f \) if there exists an interval \((x_1, x_2)\) that contains \( a \) such that

\[
x_1 < x < x_2 \quad \text{implies} \quad f(a) \leq f(x).
\]

A function value \( f(a) \) is called the relative maximum of \( f \) if there exists an interval \((x_1, x_2)\) that contains \( a \) such that

\[
x_1 < x < x_2 \quad \text{implies} \quad f(a) \geq f(x).
\]

**Average Rate of Change:** The average rate of change between any two points \((x_1, f(x_1))\) and \((x_2, f(x_2))\) is the slope of the line through the two points.

\[
\text{A. R. of C. of } f \text{ from } x_1 \text{ to } x_2 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.
\]

**Even and Odd Functions**

A function \( y = f(x) \) is even if, for each \( x \) in the domain of \( f \),

\[
f(x) = f(-x).
\]

A function \( y = f(x) \) is odd if, for each \( x \) in the domain of \( f \),

\[
-f(x) = f(-x) \quad \text{or} \quad f(x) = -f(-x).
\]
Problem 1. Use the graph of the function to find the domain and range of $f$.

Problem 2. Find the zeros of the function algebraically.

a) $f(x) = 2x^2 - 7x - 30$

b) $f(x) = \sqrt{2x} - 1$

c) $f(x) = 9x^4 - 25x^2$
Problem 3. Determine the intervals over which the function is increasing, decreasing, or constant.

a) \( f(x) = x^2 - 4x \)

b) \( f(x) = |x - 1| + |x + 1| \)

c) \( f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases} \)
Problem 4. Find the average rate of change of the function from $x_1$ to $x_2$.

a) $f(x) = x^2 - 2x + 8$, $x_1 = 1$, $x_2 = 5$.

b) $f(x) = -\sqrt{x + 1} + 3$, $x_1 = 3$, $x_2 = 8$.

Problem 5. Determine whether the function is even, odd, or neither. Then describe the symmetry.

a) $h(x) = x^3 - 5$

b) $f(t) = t^2 + 3t - 4$

Homework: Read section 1.5, do #7, 13, 15, 21, 23, 33, 35, 41, 51, 63, 71 (the quiz for this section will be taken from these problems)