10) How many 8-bit strings contain six or more 1s?

We need to consider 3 different cases. In each case, the 8-bit string is entirely determined by the location of the 1s.

Case 1: there are six 1s. We need to find how many ways there are to arrange 6 1s among 8 positions. This is $C(8,6)$.

Case 2: there are seven 1s. Following the same logic as in case 1, there are $C(8,7)$ strings that contain seven 1s.

Case 3: there are eight 1s. There's only one 8-bit string that has eight 1s, since $C(8,8) = 1$.

Since there is no overlap between these three cases, the addition principle is used to determine the total number of 8-bit strings that contain at least six 1s:

$$C(8,6) + C(8,7) + C(8,8) = 28 + 8 + 1 = 37$$

11) How many arrangements are there of all the letters in the word "rearrangement"?

Since there are repeated letters, we use the formula found on p. 422. There are 13 letters and 7 "types" of letter (3 r, 3 e, 2 a, 2 n, 1 g, 1 m, 1 t).

The number of arrangements is

$$\frac{13!}{3!\cdot 3!\cdot 2!\cdot 2!\cdot 1!\cdot 1!\cdot 1!}$$

$$= C(13,3)\cdot C(10,3)\cdot C(7,2)\cdot C(5,2)\cdot C(4,1)\cdot C(3,1)\cdot C(2,1)\cdot C(1,1) = 43{,}243{,}200$$

12) How many different 8-digit numbers can be formed using the digits in the number $25,558,999$?

This is similar to #11. The number of 8-digit numbers that can be formed is

$$\frac{8!}{3!\cdot 3!\cdot 1!\cdot 1!} = C(8,3)\cdot C(5,3)\cdot C(2,1)\cdot C(1,1) = 1{,}120$$

13) How many different assortments of one dozen donuts can be purchased from a bakery that makes donuts with chocolate, vanilla, cinnamon, and glazed icing?

This is a “selections with repetition” problem, as seen on p. 423. There are 12 *s*elections (a dozen donuts) and 4 *t*ypes of donut. The total number of assortments is then

$$C(s+t-1, s) = C(12+4-1, 12) = C(15, 12) = 455$$

You can also use the x’s and bars argument: to divide the donuts into 4 groups, you need 3 bars, so there are 15 total “objects” (objects being x’s and bars)*. The question is then, “how many ways can I arrange 12 x’s (the 12 donuts) among 15 total objects?” The answer is $C(15, 12)$.

*For example, if you wanted to make a table that represents the donut situation, you could do the following:

Chocolate	Vanilla	Cinnamon	Glazed
xxxx	xx	x	xxxxx
	x	xxxxxxxx	xxx

In this table, the first row with x’s could be written xxxx|xx|x|xxxxx, which represents a dozen donuts consisting of 4 chocolate, 2 vanilla, 1 cinnamon, and 5 glazed. The second one could be written as |x|xxxxxxxx|xxx, which is no chocolate, 1 vanilla, 8 cinnamon, and 3 glazed. The total number of ways to arrange all the x’s among the 15 positions is the total number of assortments we can have.

14) In how many different ways can 8 identical pieces of construction paper be distributed to 4 children so that each child receives at least one piece?

Following example 8.31 on p. 425, first give one piece of paper to each child. This leaves 4 sheets of paper to distribute among 4 children. This is the same type of problem as the donut one, where there are 4 selections (the 4 pieces of paper) and 4 types (the 4 children). So the total number of ways to distribute the 4 pieces of paper is then

$$C(4+4-1, 4) = C(7, 4) = 35$$

Again, a table with a couple of possible distributions might look like this:

Child 1	Child 2	Child 3	Child 4
xx	x		x
		xxxx	

The first case can be written xx|x||x (child 1 gets two pieces and child 2 and 4 get one piece while child 3 gets none (this is only for the four remaining pieces; we assume each child already had the one piece we gave them). Case 2 on the table can be written ||xxxx| and means child 3 gets all four pieces. Here we need to find out how many ways to arrange the 4 x’s from among the 7 possible positions.

15) How many positive integers less than 1000 are such that the sum of their digits is 9?

Here we think of the types of object as “places” in a 3-digit number (the ones place, the tens place, the hundreds place). So there are 3 types. Since the sum of the three digits is 9, there are 10 types of number that we can use (all the digits from 0 to 9). The total number of 3-digit numbers whose digits add up to 9 is then $C(9+3-1,9) = C(11,9) = 55$.

Here, a table with a couple of possible numbers might look like this:

Hundreds	Tens	Ones
xxxxx	xxx	x
	xxxxxxx	xx

Here the first example could be written xxxxx|xxx|x and would represent the number 531. The second one could be written |xxxxxxx|xx and would represent the number 72.

16) If a pair of dice is rolled, what is the probability that the sum of the spots that appear is 10?

The set S, the sample space, is the set of all possible outcomes of rolling a pair of dice. The number of elements of S is $|S| = 6 \cdot 6 = 36$ (two dice with 6 possible outcomes per die). The set E represents the set of all outcomes where the total number of spots showing on the dice is 10. There are only 3 ways this can happen (die 1 shows 6 and die 2 shows 4; die 1 shows 4 and die 2 shows 6; both dice show 5), so the probability of E occurring is

$$P(E) = \frac{3}{36} = \frac{1}{12}$$

17) If 5 coins are tossed, what is the probability that exactly 3 of them land tails?

The set S is the set of all possible outcomes from tossing 5 coins. The number of elements of S is $|S| = 2^5 = 32$ (since there are 2 possible outcomes per coin and there are 5 coins, so $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$). The event space is the set of outcomes where exactly 3 of the coins land tails. If 3 coins land tails, then 2 must land heads. The total number of ways this can happen is the total number of ways to arrange 3 object among 5 places, so $|E| = C(5,3)$. The probability of E occurring is then

$$P(E) = \frac{C(5,3)}{2^5} = \frac{5}{16}$$

18) If 3 persons are chosen at random from a set of 5 men and 6 women, what is the probability that 3 women are chosen?

The set S is the set of all possible combination of 3 people that can be chosen from 11 people, so $|S| = C(11,3)$. The set E is the set of all possible combinations of 3 people that can be chosen from among 6 people, so $|E| = C(6,3)$. The probability of E occurring is

$$P(E) = \frac{C(6,3)}{C(11,3)} = \frac{4}{33}$$

19) Compute the probability of being dealt each of the given hands if 5 cards are dealt from an ordinary 52-card deck.

a) Full house (3 cards of one rank, 2 cards of another rank).

b) Two pair (2 cards of one rank, 2 of another rank, and 1 of a third rank).

Please see the poker probabilities wiki for an explanation of how to answer these questions. Note that they use different notation, so $C(n,r)$ is written as $\binom{n}{r}$.

Remember that on the actual exam, the numerical answers are not important. What I want to see is that you know who to *find* the numerical answers using the appropriate formulas. Consequently, you won't be graded at all on numerical answers, so you don't even have to put them on the exam. (However, if you do put a numerical answer that is incorrect even though your formula is correct, you will lose a small amount of points).