Section 2.3

Antisymmetric property
A relation $R$ on a set $S$ is called antisymmetric if, whenever $x R y$ and $y R x$ are both true, then $x = y$.

Partial Order
A relation $R$ on a set $S$ is called a partial order if it has the following properties: $R$ is reflexive, antisymmetric, and transitive.

Hasse Diagram
To construct a Hasse diagram for the partial order $R$ on set $S$, we represent each element of $S$ by a point, and for each pair of distinct elements $x$ and $y$ in $S$, we draw an arrow from the point representing $x$ to the point representing $y$ whenever $x R y$ and there is no $s \in S$ other than $x$ and $y$ such that $x R s$ and $s R y$. Finally, we arrange each arrow so that its initial point is below its terminal point and remove all the arrowheads. Thus, by convention, a Hasse diagram is read from bottom to top, so that all the line segments between points are regarded as pointing upward.

Problem 1. Determine whether the given relation $R$ is a partial order on the set $S$. Justify your answer.

a) $S = \{1, 2, 3\}$ and $R = \{ (1, 1), (1, 2), (2, 2), (2, 3), (3, 3), (3, 1), (1, 3), (3, 2) \}$.

b) $S = \{\emptyset, \{1\}\}$, and $R = \{ (\{1\}, \emptyset), (\{1\}, \{1\}), (\emptyset, \emptyset) \}$.

c) $S$ is the collection of all subsets of $\{1, 2, 3\}$, and $A R B$ if $A \subseteq B$ or $B \subseteq A$.

d) $S$ is the collection of all subsets of $\{1, 2, 3\}$, and $A R B$ if $|A| \leq |B|$. 
Problem 2. Determine a Hasse diagram for the given partial order $R$ on set $S$.

a) $S = \{1, 2, 3, 4\}$, and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (2, 4), (3, 1)\}$.

b) $S$ is the collection of all subsets of $\{1, 2, 3, 4\}$ with an even number of elements, and $A R B$ if $A \subseteq B$.

c) The “is a subset of” relation ($\subseteq$) on the collection $S$ of all subsets of $\{1, 2, 3\}$.

Problem 3. Construct the partial order $R$ on the set $S$ from the given Hasse diagram.
Problem 4. Identify the minimal and maximal elements of $S$ with respect to the given partial order $R$.

a) $S$ is the set of nonempty subsets of $\{1, 2, 3\}$, and $A R B$ if $B \subseteq A$.

b) $S$ is the set of all real numbers $x$ such that $0 \leq x \leq 1$, and $x R y$ if $x \leq y$.

Problem 4. Let $S = \{3, 5, 12, 60, 84, 120, 168, 220\}$ and $x R y$ if $x$ divides $y$.

a) Show that $R$ is a partial order on the set $S$.

b) Determine a Hasse diagram. Identify minimal and maximal elements.

Homework: Read section 2.3, do #1-19 odd, 14.