

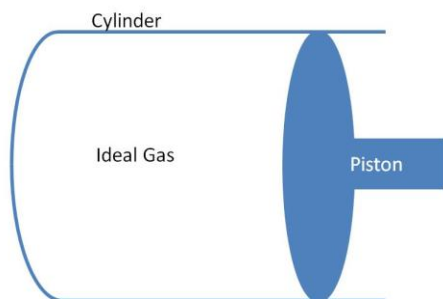
Application of the Chain Rule in an Application of the Ideal Gas Law

Background: In Chemistry, the Ideal Gas Law plays an important role. It states the relationship between the pressure, volume, temperature and number of moles of a gaseous material. The Ideal Gas Law can be stated as

$$PV = nRT$$

where P = pressure, V = volume, n = number of moles of an ideal gas, T = temperature, and R = constant.

Experiment: Suppose a sample of n moles of an ideal gas is held in an isothermal (constant temperature, T) chamber with initial volume V_0 . The ideal gas is compressed by a piston as shown in the diagram.



As the gas is compressed, the volume decreases. According to the Ideal Gas Law, the pressure will increase.

Let us assume that the volume of the gas changes at a constant rate as defined by the equation

$$V(t) = V_0 - kt,$$

where t is the time and k is a constant.

Problem Statement: Use the chain rule can to find the time rate of change of the pressure with respect to time.

Solution: The ideal gas law can be solved for the pressure, $P(t)$ to give:

$$P(t) = \frac{nRT}{V(t)},$$

where $P(t)$ and $V(t)$ have been written as explicit functions of time and the other symbols are constant. Let us differentiate each side of the equation with respect to time.

$$\frac{dP(t)}{dt} = \frac{d}{dt} \left(\frac{nRT}{V(t)} \right)$$
$$\frac{dP(t)}{dt} = nRT \frac{d}{dt} \left(\frac{1}{V(t)} \right),$$

where the constant terms, n , R , and T , have been moved to the left of the derivative operator.

Applying the chain rule gives

$$\frac{dP}{dt} = nRT \frac{d}{dV} \left(\frac{1}{V(t)} \right) \frac{dV}{dt} = nRT \left(-\frac{1}{V^2} \right) \frac{dV}{dt},$$

where the power rule has been used to differentiate the term $(1/V)$.

Now consider the volume

$$V(t) = V_0 - kt$$

and its derivative

$$\frac{dV}{dt} = -k$$

Substituting in for dV/dt and V yields dP/dt .

$$\frac{dP}{dt} = nRT \left(-\frac{1}{V^2} \right) (-k)$$
$$= nkRT \left(\frac{1}{(V_0 - kt)^2} \right)$$

This result represents the rate of change of pressure with respect to time.