Transient Response of an RC Circuit (Exponential Decay)

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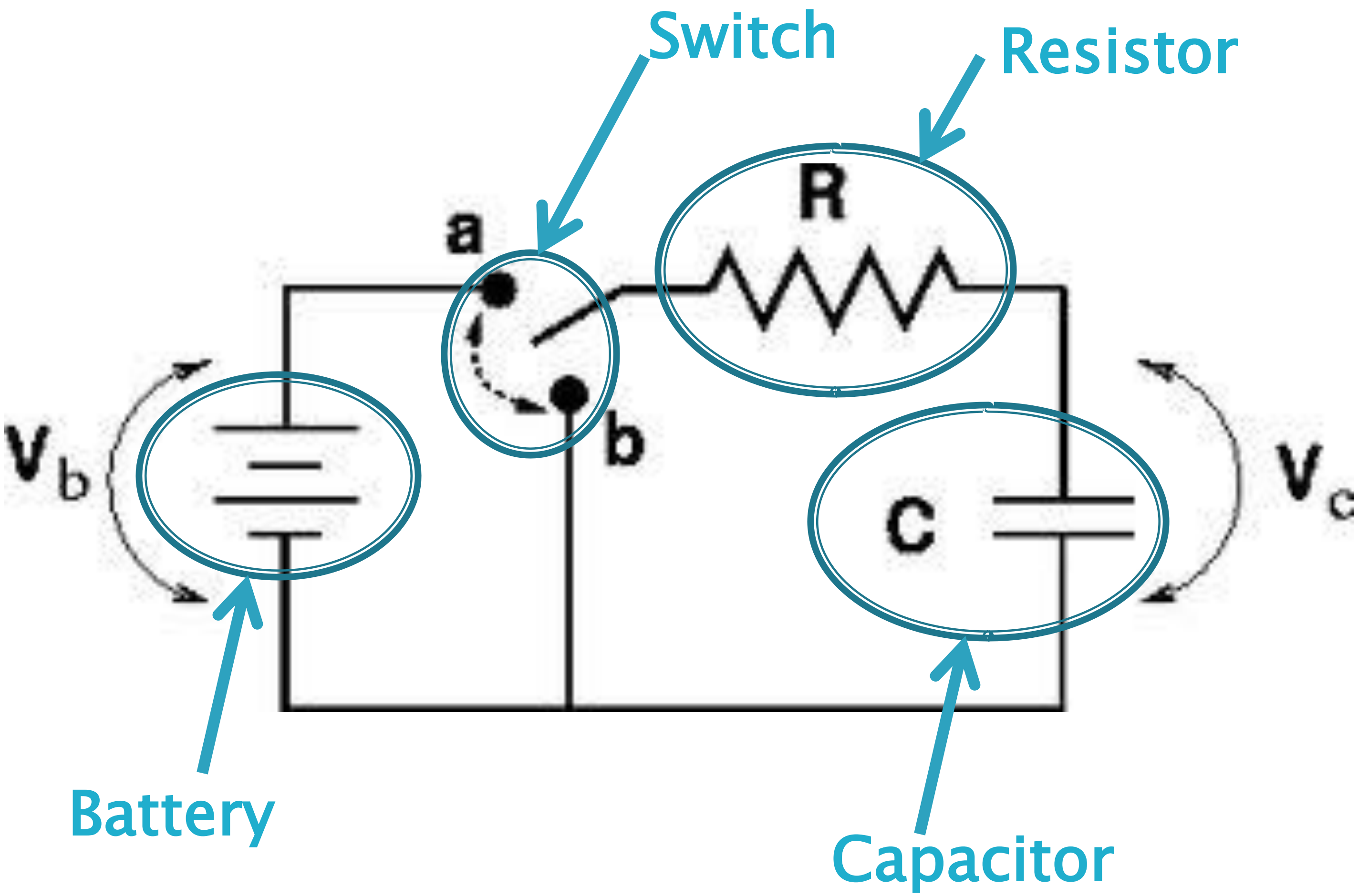
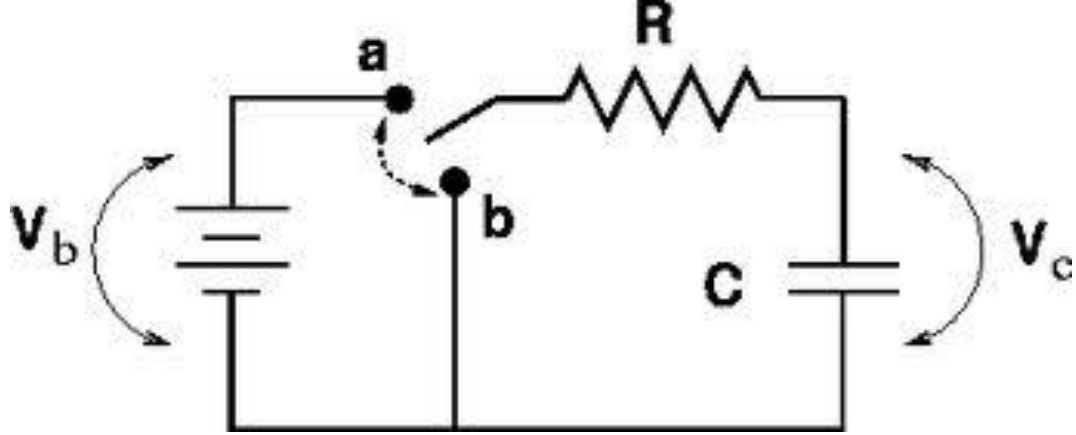
Switched RC Circuit Diagram



Description of the Components

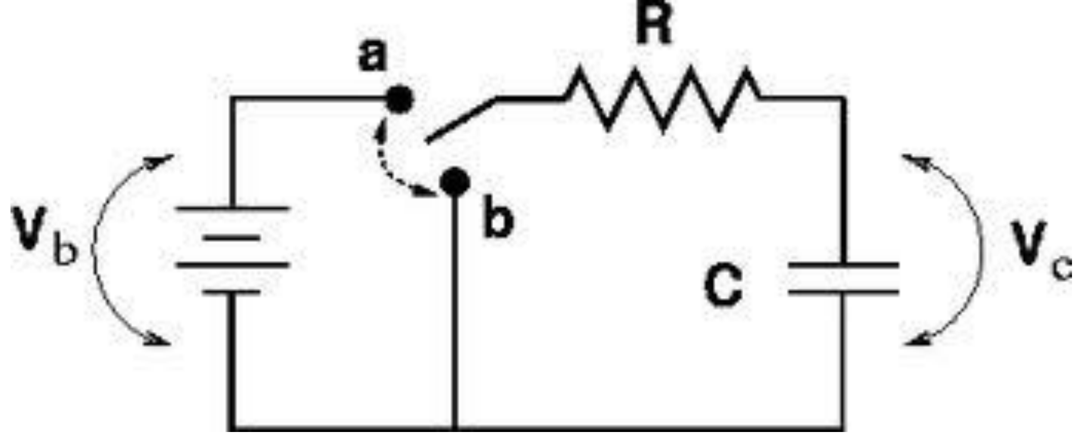
- V_b : Battery (say, 12 V battery you could buy at the store.)
- R : Resistor (electrical component that is commonly used to regulate the amount of current that flows in a circuit.
- C : Capacitor (electrical component that stores energy in an electric field.)
- V_c : Voltage across the capacitor

Switch ($t \leq 0$)



- Suppose that for time, $t \leq 0$, the switch is at position a.
- The battery "charges" the capacitor.
- The value of the capacitor voltage, V_c will be the same as the voltage of the battery, 12 V.

Switch (t>0)



- Suppose that for time, $t > 0$, the switch is moved instantaneously to position b.
- The value of the capacitor voltage, V_c , cannot change instantaneously.
- Instead, the capacitor voltage will "decay exponentially", according to the formula

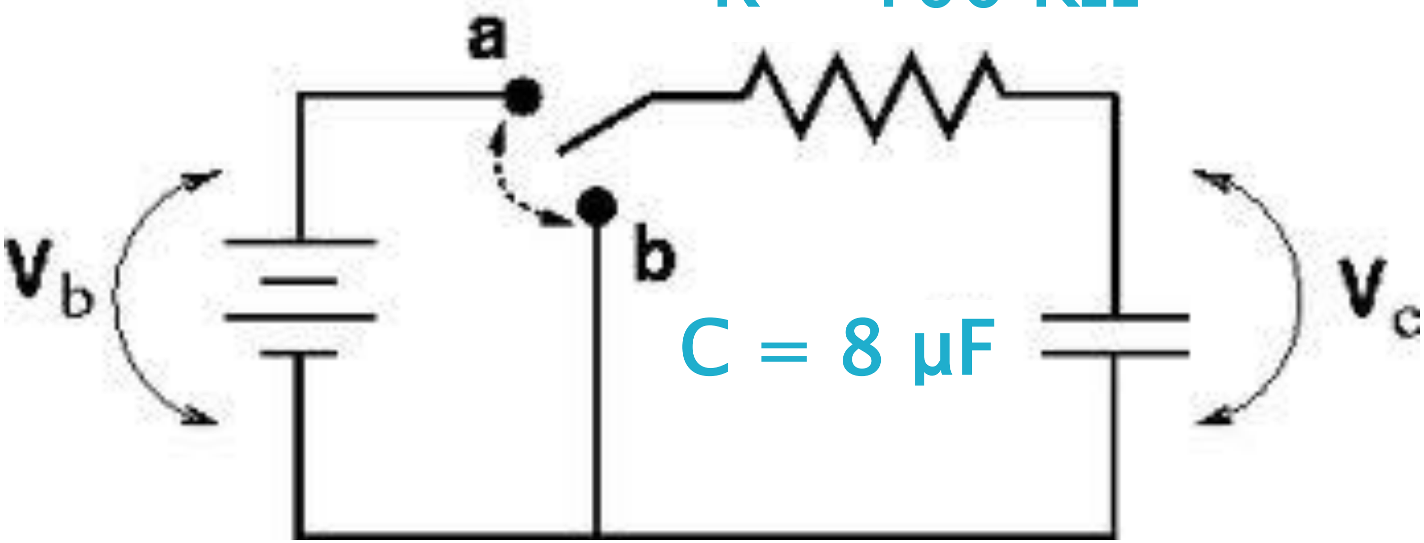
$$V_C = V_b\, e^{-\left(\frac{t}{RC}\right)}$$

Exponential Decay

- The capacitor voltage will decay from a maximum value of 12 V at time $t = 0$, to a value of 0 as t approaches ∞.
- The values of R and C will determine how quickly the decay will occur.
- The product, $\tau = RC$, is called the "time constant" of the circuit.
- One may control how quickly the voltage decays by choosing different values for the "time constant".

Let's Use Actual Values for R and C

R = 100 KΩ



$R = 100\ K\Omega = 100 \times 10^{3}\ \Omega$

$C = 8\ \mu F = 8 \times 10^{-6}\ F$

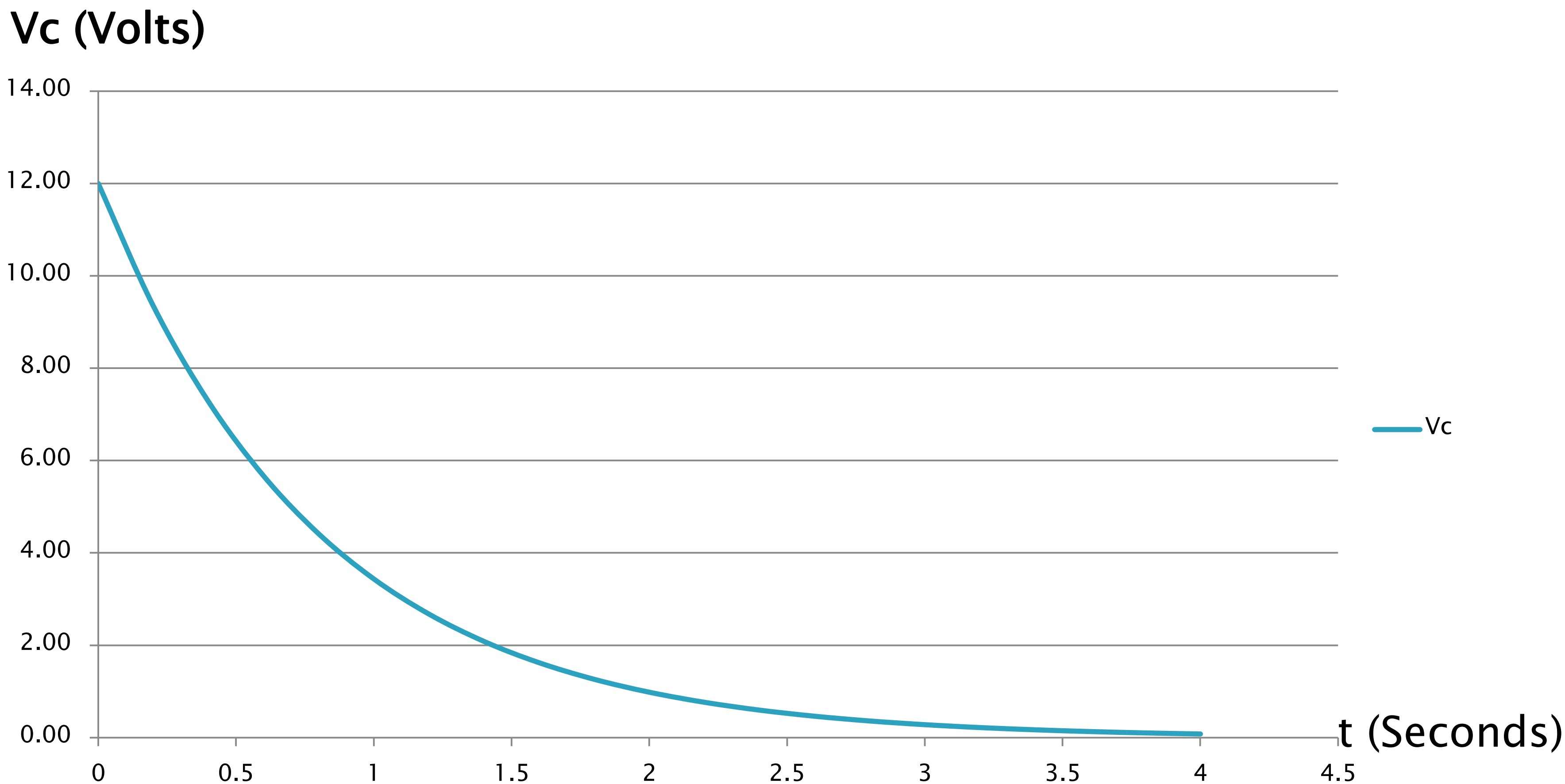
$\tau = (100 \times 10^{3})(8 \times 10^{-6}) = 0.8$ seconds

$\tau = 800$ ms

Tabular Representation

t	Vc
0	12.00
0.2	9.35
0.4	7.28
0.6	5.67
0.8	4.41
1	3.44
1.2	2.68
1.4	2.09
1.6	1.62
1.8	1.26
2	0.99
2.2	0.77
2.4	0.60
2.6	0.47
2.8	0.36
3	0.28
3.2	0.22
3.4	0.17
3.6	0.13
3.8	0.10
4	0.08

Plot of V_C



Vc (Volts)
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
t (Seconds)
Vc

Time Constant

- We found the value for the time constant for our circuit to be **$\tau = 800$ ms.**
- The formula for the capacitor voltage is

$$V_C = 12\ e^{-(t/RC)} = 12\ e^{-(t/\tau)} = 12\ e^{-(t/0.8)}$$

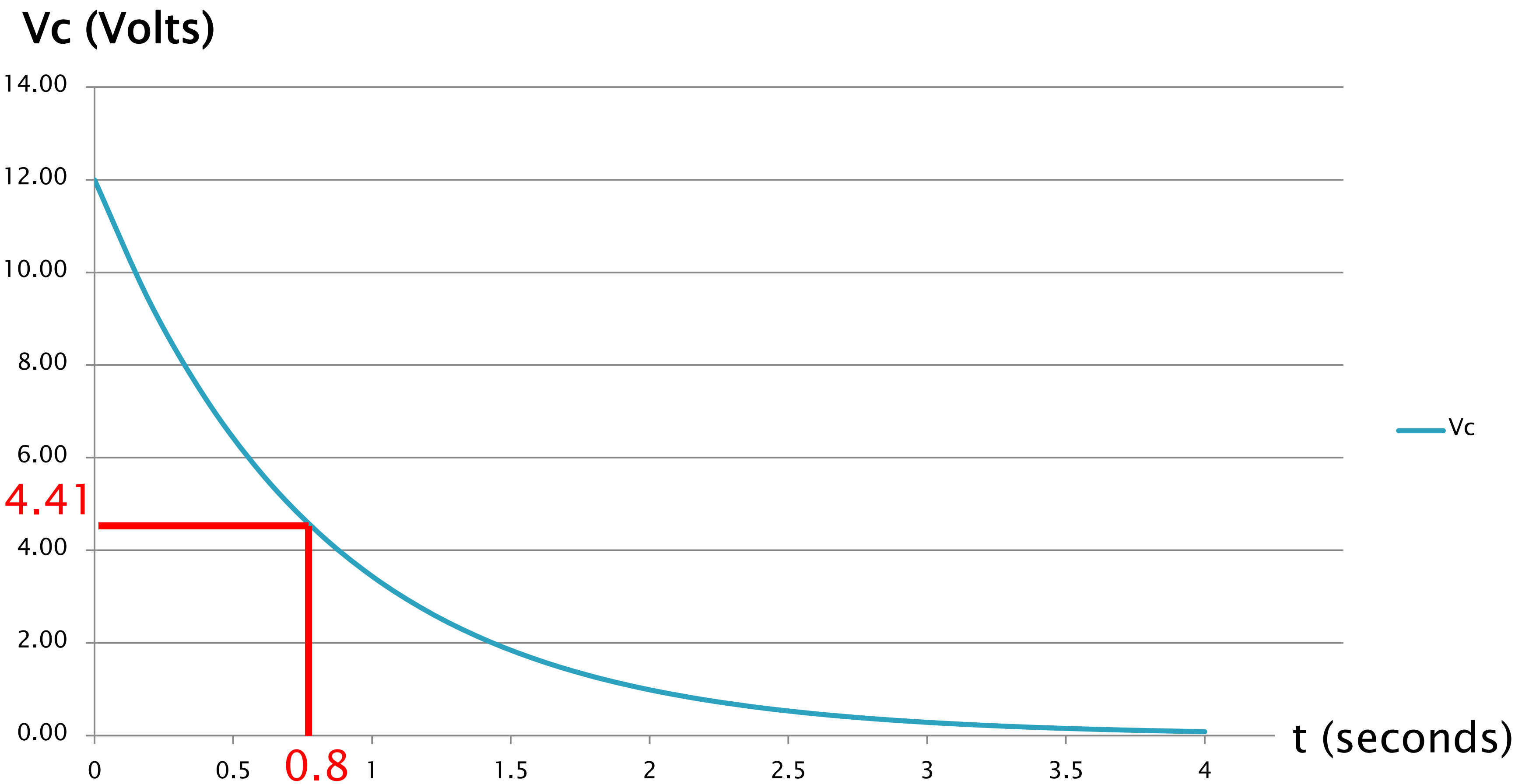
- When $t =$ **$\tau = 0.800$ s,** the capacitor voltage

$$V_C = 12\ e^{-(0.800/0.800)} = 12\ e^{-1}$$

$$= 12(0.368) = 4.41 \text{ V}$$

- The time constant tells us **how long** it takes for the voltage to decay to 36.8% of its original value.

Time Constant Shown



Summary

- Exponential decay occurs in applications in a variety of fields in science and engineering.
- We have shown an example taken from physics and electrical engineering, involving the capacitor voltage of a switched RC circuit.
- One may “control” the rate at which the decay occurs by varying the values of the resistor and the capacitor, thus influencing the time constant of the circuit.