Math 3323 | **MATRIX ALGEBRA** | Fall 2009
Dr. Duval | **Investigation 4**
due Monday, November 2

How to Use the "Vectors – Collection Linear Combinations" Module

This module is designed to display vectors and vector operations from a geometrical perspective. The module is divided into three sections, each providing different tools. This assignment will just use all three sections. The only new section is For Collection of Linear Combinations, but instructions for the other two sections are repeated for completeness.

Under the For Collection of Linear Combinations, values of a, b and c are entered. The module uses the values of a, b and c to produce the geometric representation (as dots) of the collection of all linear combinations of the vectors, $nk + ml + si + sj$ where k, l, i, and j are the vectors and the values of n range from $-a$ to a; the values of m range from $-b$ to b and the values of s range from $-c$ to c.

Under the Vectors section, vectors are associated with the names i, j, k, and l. [Here, i, j, and k do **not** represent the unit coordinate vectors.] Any vector can be entered including the stored vectors labeled $a0 - a14$. The vector $a0$ is the zero vector. Each vector is represented by a thick line segment with a color code. The module runs by Mathematica software. In Mathematica, vectors are entered using curly set notation. For instance, Mathematica recognizes $\{1, 2, 3\}$ as a vector. The red dot is the origin (the point $\{0, 0, 0\}$).

Under the Construction of Single Linear Combination section, you can assign values for the symbols d and e. These values produce thin lines which are parallel copies (with the same colors as the vectors represented) of scalar multiples of the pair of vectors k and l. For instance, when the values $d = 2$ and $e = 3$ are entered, the program produces thin lines with the same color as the vectors k and l, providing the length of the lines as twice the first vector and three times the second vector respectively. This section also has a window to enter new vectors under the name w.

General graphing commands: Hitting the Graph button will graph whatever values you have specified. Once the graph is made, pressing the Shift button (on your keyboard) and dragging the mouse will resize the graph. Moving the mouse will rotate the graph.

Assignment

Before each question, make sure to reset each vector to the zero vector namely $a0$ and each value to 0.

1. Enter the vectors $u = \{1, 2, 3\}$, $v = \{1, 0, 6\}$, $t = \{3, 4, 12\}$ for k, l, and i. Make sure the j vector is set to a0. Run the module (click Graph).
   (a) Use the module to find (if they exist) the values of c_1 and c_2 such that $t = c_1u+c_2v$. (Values for c_1 and c_2 can be found by trying values for e and d options of the module; use e and d options for values of c_1 and/or c_2. Remember the role of e and d from the previous homework and the description above). Use the c_1 and c_2 values to state a solution(s) for the **vector equation** $d_1u + d_2v + d_3t = \mathbf{0}$ ($\mathbf{0}$ represents the zero vector $\{0, 0, 0\}$). State the values of d_1, d_2 and d_3 that form solutions for the **vector equation**. How many solutions would the **vector equation** have? Explain.
   (b) For the same vectors as in part (a), enter the value 2 for a, b and the value 1 for c (you can change the values of a, b, and c to obtain more, or fewer, points). Enter $e_1u + e_2v + e_3t$ for the w vector, using the values $e_1 = 1$, $e_2 = 1$, and $e_3 = 0$ (that is, the vector is $w = 1\{1, 2, 3\} + 1\{1, 0, 6\} + 0\{3, 4, 12\}$ for the values $e_1 = 1$, $e_2 = 1$, $e_3 = 0$). Run the module. What did you observe? Explain. Repeat the process this time using first the values of $e_1 = 1$, $e_2 = -1$ and $e_3 = 1$, and next using other values of e_1, e_2 and e_3 that you choose. Run the module. What do you observe? Explain. Make a table of all the values of e_1, e_2 and e_3 you used and for each group of values, state your observations on the same table.
   (c) Based on your work on parts (a) and (b), what would you say about the sets $\{u, v\}$, $\{u, t\}$ and $\{u, v, t\}$? Explain.

2. First reset the vectors to a0 and values to 0 in the module, and run the module. Now, repeat parts (a)–(c) from question 1, but this time with the vectors $u = \{1, 0, 6\}$, $v = \{3, 4, 0\}$ and $t = \{1, -2, -5\}$. Make sure the j vector is set to a0. For this set of vectors, it may be easier to start with part (b) before part (a). Also, in part (b) for this question, start with the value 0 for c.

3. First reset the vectors to $a0$ and values to 0 in the module, and run the module. Now, repeat parts (a)–(c) from question 1 for the vectors $u = a1$, $v = a5$, and $t = a8$.

4. Now we will move up to 4 vectors. First reset the vectors to $a0$ and values to 0 in the module and run the module. Enter the vectors $a2$, $a3$, $a4$, and $a5$ into the module. In order to get different perspectives, you may need to enter the vectors in differing orders; that is, in order to apply d and e for as many vectors, one can enter first $a4$ and $a5$ as k and l ($a3$ and $a2$ as i and j); next enter $a3$ and $a2$ as k and l. It's up to you to enter the vectors in the order that's the most helpful to you, and you can run the experiments with the vectors in several different orders (just don't get confused which color is $a2$, which is $a3$, etc.).

(a) Find and state as many values for c_1, c_2, and c_3 as you can so that $a3 = c_2a2 + c_4a4 + c_5a5$. Use all the values for c_2, c_4 and c_5 to state a solution(s) for the **vector equation** $d_2a2 + d_3a3 + d_4a4 + d_5a5 = \mathbf{0}$ ($\mathbf{0}$ again represents the zero vector $\{0, 0, 0\}$). State all the values of d_2, d_3, d_4 and d_5 that form solutions for the **vector equation**. Can you find values for c_3', c_4' and c_5' such that $a2 = c_3'a3 + c_4'a4 + c_5'a5$? Explain your answer.

(b) For the same vectors as in part (a) of this question, enter $a = 2$, $b = 2$, and $c = 0$ (you can change values of a, b, and c to get more, or fewer, points). Enter $e_1a_4 + e_2a5 + e_3a3 + e_4a2$ for the w vector, first using the values $e_1 = 1$, $e_2 = 1$, $e_3 = 1$, and $e_4 = 0$, next using the values $e_1 = 1$, $e_2 = 1$, $e_3 = -1$ and $e_4 = -1$, finally using other values of e_1, e_2, e_3 and e_4 that you choose. What did you observe? Explain. State the values of e_1, e_2, e_3, e_4 you used. Now, for $a = 2$, $b = 2$, and $c = 1$, repeat the question. What did you observe? Explain.

(c) Based on your work on parts (a) and (b), what would you say about the sets $\{a4, a5\}$, $\{a4, a5, a3\}$, $\{a4, a5, a3, a2\}$ and $\{a4, a5, a2\}$? Explain.

5. State the type(s) of solution set for a **vector equation** with:
   (a) 2 vectors;
   (b) 3 vectors;
   (c) 4 vectors;
   (d) any number of vectors greater than 4 in R^3.

   Explain and justify your answers. You may need to try your own sets with 2, 3, 4 vectors to be able to make sound decisions.

6. Overall during this activity what did you observe and learn about vectors, vector equations, and their solutions? Explain.