Math 4326 **LINEAR ALGEBRA** Spring 2008
Dr. Duval **Homework**

Thursday, February 7

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Null Spaces and Ranges (part II)

pp. 45–47

A: Reading questions. Due by 2pm, Wed., 13 Feb.

1. In the proof of Theorem 3.4, what are m and n, and how do we know $\dim V = m + n$? How do we compute $\dim \operatorname{null} T$ and $\dim \operatorname{range} T$? [Note: Theorem 3.4 is the most important theorem of the first four chapters of the book, and also has one of the longest proofs in these chapters. You can answer these reading questions just from carefully reading and understanding the first paragraph of the proof, which is all I ask you to do, though, of course, you are welcome to read the rest of the proof.]
2. In the proof of Corollary 3.5, there is a string of equalities and inequalities. The middle line of this string reads "$\geq \dim V - \dim W$". Explain why "$\geq$" is the correct relation here.
3. Near the top of p. 47, the text claims "we can rewrite the equation $Tx = 0$ as a system of homogeneous equations". Why?

B: Warmup exercises. For you to present in class. Due by end of class Thu., 14 Feb.

Ch. 3: Exercises 9, 12

The Matrix of a Linear Map (part I)

pp. 48–50

A: Reading questions. Due by 2pm, Mon., 18 Feb.

1. In the middle of p. 49, the textbook offers an example corresponding to a 3-by-2 matrix. Make up your own example corresponding to a 2-by-4 matrix, with no 0 entries, and all entries being different. Now explain your example just as carefully as the textbook explains its example. You may use the textbook's example as a template for your own.
2. Verify equations 3.9 and 3.10.
3. Near the bottom of p. 50, the text claims that $\mathrm{Mat}(m, n, \mathbf{F})$ is a vector space, and asks you to verify this. Though you may want to check all the properties of a vector space (see p. 9) for yourself, please just turn in a verification of the **first** distributive property on p. 9.

B: Warmup exercises. For you to present in class. Due by end of class Tue., 19 Feb.

Differentiation Pick bases of $\mathcal{P}_m(\mathbf{F})$ and $\mathcal{P}_{m-1}(\mathbf{F})$. [Hint: Pick nice bases!] Write out $\mathcal{M}(T)$, where $T \in \mathcal{L}(\mathcal{P}_m(\mathbf{F}), \mathcal{P}_{m-1}(\mathbf{F}))$ is the differentiation linear map, *i.e.*, $Tp = p'$. If doing this for arbitrary m is too hard, try the special case where $m = 5$ first.

Multiplication by x^2 Repeat for $T \in \mathcal{L}(\mathcal{P}_m(\mathbf{F}), \mathcal{P}_{m+2}(\mathbf{F}))$, the multiplication by x^2 linear map, *i.e.*, $T(p(x)) = x^2 p(x)$.