Math 4326 | **LINEAR ALGEBRA** | Spring 2008

Dr. Duval | **Homework**

Thursday, March 13

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Diagonal Matrices

pp. 87–90

A: Reading questions. Due by 2pm, Wed., 19 Mar.

1. Verify the claim at the bottom of p. 87 that an operator $T \in \mathcal{L}(V)$ has a diagonal matrix (with $\lambda_1, \ldots, \lambda_n$ on the diagonal and 0's elsewhere) with respect to a basis $(v_1, \ldots, v_n)$ of V if and only if
$$Tv_1 = \lambda_1 v_1; \ldots ; Tv_n = \lambda_n v_n.$$
2. Verify that the only eigenvalue of the linear operator T given in equation 5.19 is 0, and that the corresponding set of eigenvectors is only 1-dimensional. In what way is this surprising?
3. Verify that $T \in \mathcal{L}(\mathbf{F}^3)$ defined by $T(z_1, z_2, z_3) = (4z_1, 4z_2, 5z_3)$, as on p. 88, satisfies each of the five conditions (a)–(e) of Proposition 5.21. You may need to read the proof to help you with some of these, though I don't think you need to fully understand the proof in order to complete the verification of (a)–(e).

B: Warmup exercises. For you to present in class. Due by end of class Thu., 20 Mar.

Ch. 5: Exercise 11.

Inner Products

pp. 98–101

A: Reading questions. Due by 2pm, Mon., 31 Mar.

1. The text claims near the bottom of p. 98 that "[t]he norm is not linear on $\mathbf{R}^n$." Verify this claim. [Hint: Define a function $T\colon \mathbf{R}^n \to \mathbf{R}^n$ by $N(x) = \|x\|$, and show N is not linear.] How does this claim relate to the introduction of inner products?
2. Provide a little more explanation for the claim near the bottom of p. 99, "The equation above thus suggests that the inner product of $w = (w_1, \ldots, w_n) \in \mathbf{C}^n$ with z should equal
$$w_1\bar{z}_1 + \cdots + w_n\bar{z}_n."$$
3. Match the properties of the dot product described at the bottom of p. 98 to the five properties listed at the top of p. 100 that define an inner product.
4. Provide justification for each step in the derivation, on p. 101, that $\langle u, v + w\rangle = \langle u, v\rangle + \langle u, w\rangle$. Note that some of these will be properties of inner products, and others will be properties of complex conjugates (see p. 69).

B: Warmup exercises. For you to present in class. Due by end of class Tue., 1 Apr.

Verify 6.1 and 6.2 define inner products, as claimed.