Math 4326 **LINEAR ALGEBRA** Spring 2015
Dr. Duval **Homework**
Monday, March 23

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Eigenvectors and Upper-Triangular Matrices (Part II):
Upper Triangular Matrices
Section 5.B, pp. 146–152

A: Reading questions. Due by 2pm, Sun., 29 Mar.

1. Give three examples of a 3-by-3 upper-triangular matrix.
2. The author introduces upper-triangular matrices as nice ones to represent linear transformations. Given a fixed linear transformation (in other words, you don't get to pick the linear transformation), how can you represent it by an upper-triangular matrix? In other words, *what* can you pick cleverly to make sure the fixed linear transformation is represented by an upper-triangular matrix?
3. Demonstrate parts (b) and (c), for $j = 2$, of result 5.26 (Conditions for upper-triangular matrix) on Example 5.23. In other words, show that $Tv_2 \in \operatorname{span}(v_1, v_2)$ and that $\operatorname{span}(v_1, v_2)$ is invariant under T. Note that the basis here is the standard basis.
4. In the middle of the second paragraph of the proof of result 5.27 (Over $\mathbf{C}$, every operator...), the text claims that, if λ is an eigenvalue of T, then $T - \lambda I$ is not surjective by (3.69). Fill in the missing details of this claim.
5. Let $T \in \mathcal{L}(V)$. How does finding a basis of V for which the matrix of T is upper triangular help find the eigenvalues of T? How does it help determine whether or not T is invertible? Demonstrate your answers on Example 5.23 (in other words, find the eigenvalues of that matrix, and also determine whether or not it is invertible).

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 30 Mar.

Exercises 5.B: 14, 15. [Hint: In each case, there is a 2×2 matrix.]

Eigenspaces and Diagonal Matrices
Section 5.C

A: Reading questions. Due by 2pm, Tue., 31 Mar.

1. Verify the claim in the middle of p. 155 that if an operator has a diagonal matrix with respect to a basis, then the entries along the diagonal are precisely the eigenvalues for the operator, **without** using result 5.32. In other words, find the "easier proof for diagonal matrices" suggested in that paragraph. [Hint: This just uses definitions.]
2. Verify the details of Example 5.40, as the textbook suggests you do.
3. Verify that the only eigenvalue of the linear operator T given in Example 5.43 is 0, and that the corresponding set of eigenvectors is only 1-dimensional. In what way is this surprising?
4. Near the end of Example 5.45, the textbook says "These simple equations are easy to solve". Write down these simple equations, and show how to solve them.
5. Verify that $T \in \mathcal{L}(\mathbf{F}^3)$ defined by $T(z_1, z_2, z_3) = (4z_1, 4z_2, 5z_3)$, as on p. 160, satisfies each of the five conditions (a)–(e) of result 5.41 (Conditions equivalent to diagonalizability). You may need to read the proof to help you with some of these, though I don't think you need to fully understand the proof in order to complete the verification of (a)–(e).

B: Warmup exercises. For you to present in class. Due by end of class Wed., 1 Apr.

Exercises 5.C: 7, 8