Math 4326 Dr. Duval

LINEAR ALGEBRA Homework

Monday, March 30

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all* four standard parts (a), (b), (c), (d) of Part A in what you hand in.

Inner Products and Norms (Part I): Inner Products

Section 6.A, pp. 164–168

A: Reading questions. Due by 2pm, Sun., 5 Apr.

- 1. The text claims in the middle of p. 164 that "The norm is not linear on \mathbf{R}^{n} ." Verify this claim. [Hint: Define a function $N: \mathbf{R}^{n} \to \mathbf{R}$ by N(x) = ||x||, and show N is not linear.] How does this claim relate to the introduction of inner products?
- 2. Provide a little more explanation for the claim in the middle of p. 165, "The equation above thus suggests that the inner product of $w = (w_1, \ldots, w_n) \in \mathbb{C}^n$ with z should equal

 $w_1\bar{z}_1 + \cdots + w_n\bar{z}_n$."

- 3. Match the properties of the dot product described at the bottom of p. 164 to the five properties of Definition 6.3 (inner product).
- 4. Provide justification for each step in the derivation of 6.7(d), that $\langle u, v + w \rangle = \langle u, v \rangle + \langle u, w \rangle$. Note that some of these will be properties of inner products, and others will be properties of complex conjugates (see Definition 4.3).
- B: Warmup exercises. For you to present in class. Due by the end of class Mon., 6 Apr.

Verify 6.4(a)-(c) are inner products (satisfy all five conditions of Definition 6.3)

Inner Products and Norms (Part II): Norms Section 6.A, pp. 168-174

A: Reading questions. Due by 2pm, Tue., 7 Apr.

- 1. Provide justification for each step in the derivations, of results 6.10(b) and 6.18, respectively, that ||av|| = |a|||v||, and $||u + v|| \le ||u|| + ||v||$. Note that some of these will be properties of inner products, and others will be properties of complex conjugates (see Definition 4.3).
- 2. Verify the claim in the middle of p. 171 that the displayed equation above result 6.14 writes u as a scalar multiple of v plus a vector orthogonal to v.
- 3. Directly verify the Cauchy-Schwarz inequality (result 6.15) for the following pairs of vectors:
 - (3, 1, 4) and (2, 7, 1) in \mathbb{R}^3 , with inner product 6.4(a); and
 - x^2 and 7x 2 in $\mathcal{P}_2(\mathbf{R})$, with inner product 6.4(c).

B: Warmup exercises. For you to present in class. Due by end of class Wed., 8 Apr.Exercises 6.A: 4