Math 4326 Dr. Duval

# LINEAR ALGEBRA Homework

Tuesday, January 31

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all* four standard parts (a), (b), (c), (d) of Part A in what you hand in.

### Dimension

Section 2.C

## A: Reading questions. Due by 2pm, Mon., 6 Feb.

- 1. What is the significance of result 2.35 (Basis length ...)? Why must it be the first result of this section? [Hint: What is the name of this section?]
- 2. Find the definition of "finite-dimensional" vector space in the text. [Hint: Believe it or not, it is **not** in this section!] How does it compare to the definition of "dimension" in this section? Why are these two definitions compatible?
- 3. Which do you think will prove to be more useful, Proposition result 2.39 (Linearly independent list ...), or result 2.42 (Spanning list ...)? Why?
- 4. Which parts of Example 2.41 would not work if we replaced  $(x 5)^2$  by (x 5), and why?
- 5. Verify that result 2.43 (Dimension of a sum) works when  $U_1$  is the *xy*-plane, and  $U_2$  is the *yz*-plane, in  $\mathbb{R}^3$ .

B: Warmup exercises. For you to present in class. Due by end of class Tue., 7 Feb.
Exercises 2.C: 1, 4, 11

# The Vector Space of Linear Maps Section 3.A

#### A: Reading questions. Due by 2pm, Wed., 8 Feb.

- 1. Verify the following functions, described on pp. 52–53, are in fact linear maps: identity, differentiation, multiplication by  $x^2$ , backward shift.
- 2. Let's illustrate one part of result 3.5 (Linear maps and basis ...), namely the need for  $v_1, \ldots, v_n$  to be a basis, with an example. First, explain why (1, 0, 1), (0, 1, 1), (1, 1, 2) is **not** a basis of  $\mathbf{F}^3$ . Then show that there is **no** linear map  $T: \mathbf{F}^3 \to \mathbf{F}^4$  such that T((1, 0, 1)) = (2, 0, 1, 7), T((0, 1, 1)) = (1, 9, 6, 6), T((1, 1, 2)) = (3, 9, 6, 12).
- 3. Verify that S + T is a linear map from V to W whenever  $S, T \in \mathcal{L}(V, W)$ .
- 4. Verify the first distributive property on p. 56:  $(S_1 + S_2)T = S_1T + S_2T$  whenever  $T \in \mathcal{L}(U, V)$  and  $S_1, S_2 \in \mathcal{L}(V, W)$ .
- B: Warmup exercises. For you to present in class. Due by the end of class Thu., 9 Feb.Exercises 3.A: 1, 5, 8