Math 4326 | **LINEAR ALGEBRA** | Spring 2017
Dr. Duval | **Homework**

Tuesday, April 4

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Orthonormal Bases

Section 6.B

A: Reading questions. Due by 2pm, Mon., 10 Apr.

1. Verify the lists in Example 6.24 are indeed orthonormal, as claimed in the text.
2. Demonstrate result 6.30 (Writing a vector…) with $V = \mathbf{F}^3$, orthonormal basis (e_1, e_2, e_3) given by the list in Example 6.24(c), and $v = (9, 1, 5)$.
3. Try to read the proof of the Gram-Schmidt Procedure (result 6.31) without worrying too much about the precise algebraic details of the equation defining e_j or the calculation in the middle of p. 183. The second sentence of the statement of the result says, "For $j = 2, \ldots, m$, define e_j inductively…". What, in your own words, does that mean in this case?
4. The proof of result 6.37 (Upper-triangular…) relies on the proof of the Gram-Schmidt Procedure (result 6.31), in particular claiming that applying the Gram-Schmidt Procedure to $v_1, \ldots, v_n$ produces an orthonormal basis $e_1, \ldots, e_n$ such that
$$\operatorname{span}(e_1, \ldots, e_j) = \operatorname{span}(v_1, \ldots, v_j)$$
for **each** j. Verify that the orthonormal basis e_1, e_2, e_3 in Example 6.33 satisfies this condition for each of $j = 1, 2, 3$.
5. In Example 6.44, we replace $\cos(\pi t)$ by $u(t)$ in the integral. Why is this worth doing? In other words, in what way is the expression with $u(t)$ better than the expression with $\cos(\pi t)$?

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 11 Apr.

Exercises 6.B: 5.

Orthogonal Complements and Minimization Problems

Section 6.C

A: Reading questions. Due by 2pm, Wed., 12 Apr.

1. Find $U^\perp$ for $U = \operatorname{span}((9, 1, 5))$ in $V = \mathbf{R}^3$. Describe $U^\perp$ geometrically in this case.
2. Verify result 6.47 (Direct sum…orthogonal complement) in the case of question 1 above.
3. Find $P_U v$ for $v = (1, 2, 3)$ and $U = \operatorname{span}((9, 1, 5))$ in $V = \mathbf{R}^3$.
4. In Example 6.58, approximating $\sin x$ by a 5th-degree polynomial, explain how $\int_{-\pi}^{\pi} |\sin x - u(x)|^2 \, dx$ is minimized using the inner product 6.59 and result 6.56 (Minimizing the distance…).

B: Warmup exercises. For you to present in class. Due by end of class Thu., 13 Apr.

Exercises 6.C: 4, 11.