

Thursday, April 13

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Self-Adjoint and Normal Operators (Part I):
Adjoins

Section 7.A, pp. 203–208

A: Reading questions. Due by 2pm, Mon., 17 Apr.

1. Define $T: \mathbf{R}^2 \rightarrow \mathbf{R}^4$ by

$$T(x_1, x_2) = (2x_1 + x_2, 3x_2, x_1 - x_2, -2x_2).$$

Find a formula for T^* , the adjoint of T .

2. Provide justification for every equation in the proof of result 7.5 (The adjoint is a linear map).
3. Provide justification for every equation in the proof of result 7.6(a) (Properties of the adjoint: additivity).
4. Let T be the linear map in question 1 above. Find a vector $v \in \text{null } T^*$ ($v \neq 0$), and show $v \in (\text{range } T)^\perp$, thus providing an example of result 7.7(a) (Null space and range of T^*).

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 18 Apr.

Exercises 7.A: 1

Self-Adjoint and Normal Operators (Part II):
Self-Adjoint Operators; Normal Operators

Section 7.A, pp. 209–214

A: Reading questions. Due by 2pm, Wed., 19 Apr.

1. Verify that the sum of two self-adjoint operators is self-adjoint, as the textbook suggests you do on p. 209.
2. Where in the proof of result 7.13 (Eigenvalues of self-adjoint operators are real) do we use that T is self-adjoint?
3. What is the relationship between self-adjoint and normal operators? (If an operator is self-adjoint, do you know whether it is normal? If an operator is normal, do you know whether it is self-adjoint?)
4. Verify result 7.20 (T is normal. . .) on Example 7.19.
5. Verify that if λ is a scalar and T is normal then $T - \lambda I$ is also normal, as the textbook suggests you do in the proof of result 7.21 (For T normal, T and T^* have the same eigenvectors).
6. Verify result 7.22 (Orthogonal eigenvectors for normal operators) on Example 7.19.

B: Warmup exercises. For you to present in class. Due by end of class Thu., 20 Apr.

Exercises 7.A: 7, 14