Math 4326/5322 | **LINEAR ALGEBRA** | Spring 2019
Dr. Duval | **Main Exercises 10**
due 2pm, Thursday, April 25

1. Suppose $e_1, \dots, e_n$ is a list of vectors in an n-dimensional inner product space V.

(a) Prove that if $e_1, \dots, e_n$ is orthonormal then

$$\langle a_1 e_1 + \cdots + a_n e_n,\ b_1 e_1 + \cdots + b_n e_n \rangle = a_1 b_1 + \cdots a_n b_n$$

for any $a_1, \dots, a_n, b_1, \dots b_n$.

(b) Prove the converse: if

$$\langle a_1 e_1 + \cdots + a_n e_n,\ b_1 e_1 + \cdots + b_n e_n \rangle = a_1 b_1 + \cdots a_n b_n$$

for any $a_1, \dots, a_n, b_1, \dots b_n$, then $e_1, \dots, e_n$ is orthonormal.

2. (a) Prove that

$$\langle f, g \rangle = \int_{-1}^{1} x^2 f(x) g(x) dx$$

is an inner product on the vector space $C([-1, 1])$ of continuous real-valued functions on the domain $[-1, 1]$.

(b) Use the Gram-Schmidt process to find an orthonormal basis for $\mathcal{P}_2(\mathbf{R})$ with respect to this inner product.

(c) Find a polynomial $q(x)$ such that

$$p(\frac{1}{2}) = \int_{-1}^{1} x^2 p(x) q(x) dx$$

for every $p \in \mathcal{P}_2(\mathbf{R})$.

3. (Graduate students only) What happens when the Gram-Schmidt process is applied to a list that is not linearly independent?