Math 4326/5322 | **LINEAR ALGEBRA** | Spring 2019

Dr. Duval

Main Exercises 7

due 2pm, Thursday, April 4

1. Define $T \in \mathcal{L}(\mathbf{F}^3)$ by

$$T(x_1, x_2, x_3) = (4x_3, -3x_2, 0).$$

Find all eigenvalues and eigenvectors of T.

2. Let V be a vector space, and assume that S and T are inverses of each other in $\mathcal{L}(V)$. Prove that if $v \in V$ is an eigenvector for T with eigenvalue λ, then v is also an eigenvector for S. Find the eigenvalue for v with respect to S, and prove your answer is correct.

3. Let V be a vector space, and let $T \in \mathcal{L}(V)$. Define $S \in \mathcal{L}(V)$ by $S = 2T^3 - 5T + 4I$. Prove that if $v \in V$ is an eigenvector for T with eigenvalue λ, then v is also an eigenvector for S. Find the eigenvalue for v with respect to S, and prove your answer is correct.

4. Generalize your results in question **3.** to the case where $S = p(T)$ for an arbitrary polynomial $p \in \mathcal{P}(\mathbf{R})$.

5. Let V be a vector space, and let $U_1, \ldots, U_k$ be subspaces of V. Also let $T \in \mathcal{L}(V)$. Prove that if $U_1, \ldots, U_k$ are invariant under T, then $U_1 + \cdots + U_k$ is also invariant under T.

6. (Graduate students only) Let V be a vector space, and let $\{U_\alpha \colon \alpha \in \Delta\}$ be an arbitrary collection of subspaces of V. Also let $T \in \mathcal{L}(V)$. Prove that if every U_α is invariant under T, then $\bigcap_{\alpha \in \Delta} U_\alpha$ is also invariant under T.