Math 4370 **COMBINATORICS** Spring 2012
Dr. Duval **Homework**

Thursday, January 19

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Bijective proofs

Subsection 1.4.1

[Note: The subsubsection on Catalan numbers (pp. 25–27) is optional.]

A: Reading questions. Due by 3pm, Wed., 25 Jan.

1. In Example 1.26, what would go wrong (in the counting sense) if the streets were not one-way (in other words, if we could also drive to the south or to the west sometimes)?
2. In Example 1.26, explicitly draw the path corresponding to the subset $\{1, 2, 4, 6, 9, 10\}$.
3. What is the significance in combinatorics of finding a bijection between finite sets?
4. Match the 3-step outline for proving $|S| = |T|$ on p. 24 to the text of the proof of Proposition 1.29. For instance, you could photocopy or rewrite the proof of Proposition 1.29, and show which sentences correspond to each step of the 3-step outline on p. 24.
5. (Answer this question only if you've seen bijections before; otherwise, let me know you haven't seen bijections before.) Which part of the 3-step outline on p. 24 corresponds to the function f being one-to-one? Which part corresponds to the function f being onto?

B: Warmup exercises. For you to present in class. Due by end of class Thu., 26 Jan.

1. Repeat Example 1.26 but with $X = (6, 5)$ and $Y = (5, 2)$ (leave U and V where they are).
2. Repeat Example 1.26 parts (a) and (c) only, but with $U = (5, 4)$ and $V = (6, 3)$.
3. Apply the bijection in Example 1.31 to find the internet connection arrangement corresponding to the 10-letter word *abbcaccbaa*.

Applications of basic counting principles, continued

Subsections 1.4.2, 1.4.3

A: Reading questions. Due by 3pm, Mon., 30 Jan.

1. Demonstrate the bijection in the proof of Proposition 1.35 when $n = 5$ and $k = 2$. (In other words: Figure out where P and Q are for these values of n and k. Then explicitly list the bijection described in this proof, between the paths to P and the paths to Q. "Explicitly list" means to write down all the elements of one set, and say which element of the other set each one corresponds to.)
2. In the middle of p. 28, the author describes how to prove Proposition 1.35 using a bijection with subsets instead of with lattice paths. Demonstrate this bijection when $n = 5$ and $k = 2$.
3. Demonstrate the bijection in the proof of Theorem 1.36 when $n = 4$ and $k = 1$.
4. In Example 1.39, the author uses the Division Principle. This example is much too large to illustrate the Division Principle explicitly, as in the diagram on p. 15 (we'd need 30! items in T!). So, rewrite Example 1.39 with only 4 dance steps total, 2 steps of type A, 1 step of type B, and 1 step of type C. Then illustrate this with a diagram like the one on p. 15.

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 31 Jan.

1. Reinterpret the proof Theorem 1.37 in terms of subsets of the set $[2n]$, as suggested in the comment above Example 1.38.
2. **1.10 Supplementary Exercise:** 16a
3. How many different ways can I distribute 4 orange soccer balls, 2 blue soccer balls, and 1 silver soccer ball to 7 children, one soccer ball to each child? [Note: Consider soccer balls that are the same color to be identical; consider all children to be different.] What if two of the children are friends, and must receive the same color soccer ball? What if two of the children are siblings, and must receive different color soccer balls?
4. How many different anagrams are there of the word "TELLTALE"?