Math 4370 **COMBINATORICS** Spring 2012
Dr. Duval **Homework**
Thursday, March 1

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Inclusion-exclusion: Three sets
Subsection 2.4.2

A: Reading questions. Due by 3pm, Wed., 7 Mar.

1. At the end of the canoeing story example at the beginning of this subsection, the author states that one more vital number is needed. Use Lemma 2.35 to determine which number is needed. What are the possible values of this number?
2. Solve the problem of the canoeing story example ("How many friends went on the canoe trip this year?") for each of the values (of the "vital number") you found in the previous reading question. [Note: You may want to solve the following reading question first, I'm not sure which is best to solve first.]
3. Figure 2.13 is for two sets, but it should not be hard to figure out how to draw the corresponding figure for three sets. Do this, filling in numbers that match your solution to the canoeing story example, for each of your solutions in the previous reading question.
4. Describe how symmetry is used in the solution to Example 2.36.
5. Solve Example 2.37 if Claudia arrives sixth instead of eighth.

B: Warmup exercises. For you to present in class. Due by the end of class Thu., 8 Mar.

1. There are n people (including A, B, C, D, E, and F) who will stand in a line. In how many ways can they line up, subject to each of the following restrictions (each restriction is a new problem):
   (a) A cannot stand next to B.
   (b) A cannot stand next to B, and C cannot stand next to D.
   (c) A cannot stand next to B, and B cannot stand next to C.
   (d) A cannot stand next to B; C cannot stand next to D; and E cannot stand next to F.
2. We are going to make a fruit basket of apples, oranges, and pears. The fruit basket must consist of exactly 13 pieces of fruit. In how many different ways can we do this, subject to each of the following restrictions (each restriction is a new problem):
   (a) There is no restriction.
   (b) There must be less than 10 apples.
   (c) There must be less than 10 of each kind of fruit.
   (d) There must be less than 6 of each kind of fruit.

Inclusion-exclusion: Any number of sets
Subsection 2.4.3

A: Reading questions. Due by 3pm, Mon., 19 Mar.

1. Show how the main results of each of the two previous subsections are special cases of Theorem 2.38.
2. Page 92 is devoted to the first part of the derivation of a formula for $S(n, k)$. It explicitly discusses "one-fold intersections" and "two-fold intersections", and then jumps to the general case of j-fold intersections. Explicitly describe the situation for 3-fold intersections.
3. What is the difference between the formulas in Theorems 2.40 and 2.41, and why is it there?
4. Why is $\phi(p) = p - 1$? Are there any composite (non-prime) positive integers n such that $\phi(n) = n - 1$?
5. Explicitly compute $\phi(15)$ by checking (and counting) which numbers are relative prime to 15.

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 20 Mar.

1. Let $i < n$. In how many different ways can we permute the set $[n]$ so that i is in the ith position?
2. Let $i < j < n$. In how many different ways can we permute the set $[n]$ so that i is in the ith position **and** j is in the jth position?
3. Let $i < j < n$. In how many different ways can we permute the set $[n]$ so that i is in the ith position **or** j is in the jth position?
4. In how many ways can we permute the set $[n]$ so that there is **at least** one $i < n$ so that i is in the ith position?
5. In how many ways can we permute the set $[n]$ so that there is no $i < n$ so that i is in the ith position?