Math 4370 | **COMBINATORICS** | Spring 2012
Dr. Duval | **Homework**

Thursday, March 8

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Twelvefold way

Section 2.5

A: Reading questions. Due by 3pm, Wed., 21 Mar.

1. Try to solve Problems 2.49, 2.50, 2.51, and 2.52 **before** reading the solutions. Report on what you were and were not able to solve on your own (or with classmates) before reading the solutions.
2. Relate each of the twelve problems to where (if anywhere) they were discussed previously in the textbook. Make any notes you find useful about how the problems were solved.
3. Make a 3×4 chart of the 12 problems and their solutions. Your chart should be clear and readable.

B: Warmup exercises. For you to present in class. Due by the end of class Thu., 22 Mar.

1. **2.10 Supplementary Exercise:** 40, 41, 43, 42 [Note the instructions at the top of p. 124.]

Ordinary generating functions, part 1

Subsection 3.2.1 (up through and including Example 3.5)

Note: We will return to Section 3.1 after this introduction. But you still may want to refer to Section 3.1 as you read this material.

A: Reading questions. Due by 3pm, Mon., 26 Mar.

1. Compute a_3 and a_4 of the sequence in the example about red hats at the beginning of the section. Then compute $a_1, \ldots, a_4$ if, instead, only 4 people wearing red hats start the rumor.
2. Let's try to make sense of Definition 3.3 with two concrete examples. First consider the example of the red hats, which you have computed up to a_4. The **ordinary generating function** of this sequence is a sort of "infinite polynomial" (for more precision, see the first two or three paragraphs of subsection 3.1.2) so it will not be possible to write down the whole thing. Still, just to test out the notation of this definition, write out the beginning of this generating function, up through the term corresponding to a_4.

   For the second example, recall that the Fibonacci sequence $0, 1, 1, 2, 3, 5, 8, 13, \ldots$ is defined by letting each term of the sequence be the sum of the previous two terms, starting with 0 and 1. More formally, it is the sequence $\{a_i\}_{i \geq 0}$ defined by $a_0 = 0, a_1 = 1$, and $a_i = a_{i-1} + a_{i-2}$ for $i \geq 2$. Write out the first 10 (non-zero) terms of the ordinary generating function of this sequence.
3. In Example 3.4, the author urges the reader to justify the summation in the displayed string of equations, giving the ordinary generating function of the (very easy) sequence whose every member is 1. Accept this invitation (is it still an invitation if you are "urged" instead of "invited"?) and justify the summation. (Just to be clear, this is the **last** equality, $\sum_{n \geq 0} x^n = \frac{1}{1-x}$.) [Note typographical error: The "$n = 0$" index on summation should be "$n \geq 0$".]
4. One key step of Example 3.5 relies on computing the derivative of an ordinary generating function. Identify where in subsection 3.1.2 this idea is defined.

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 27 Mar.

1. Find the ordinary generating function of the sequence $a_0 = a_1 = a_2 = a_3 = 0$ and $a_i = 1$ for $i \geq 4$. (Hint: Compare to Example 3.4.)
2. Justify each of the equalities in Example 3.5. (Hint: The author skips a step, I think, in the second equality.) [Note typographical error: The "$n = 0$" index on summation should be "$n \geq 0$".]