Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2018
Dr. Duval | **Homework 2**
due Thursday, February 1

1. Part of the multiplication table for the group $G = \{a, b, c, d, e\}$ is given below. Complete the table.

$\times$	a	b	c	d	e
a			a		
b					
c					
d				b	
e					a

2. Recall that in class we showed that the set of all six transformations of an equilateral triangle form a group. (If you don't recall that, please contact me.) Name each of the transformations (in any way that makes sense to you), and write down the multiplication table.

Define a **flip** to be any transformation that requires you to change which side of the triangle is showing (in class, this was changing from the blue spot in front to the blue spot in back, or vice versa), and define a **rotation** to be the remaining transformations. Identify which transformations in your table are flips, and which are rotations. [Hint: There should be three of each.]

Prove that the set of all rotations forms a subgroup of the group of transformations. Does the set of all flips form a subgroup? Prove your answer is correct.

Find all the subgroups of order 2 (all subgroups consisting of exactly 2 elements) in the group of transformations, and prove your answer is correct.

3. Is the following subset H of $GL(2, \mathbf{R})$ a subgroup of $GL(2, \mathbf{R})$?

$$H = \left\{ \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} : b \in \mathbf{R} \right\}.$$

Prove your answer is correct.

4. Let G be a nonempty set that is closed under an associative binary operation called multiplication. Prove that G is a group if and only if the equation $axb = c$ has a solution x for all choices of a, b, c in G.