Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2018
Dr. Duval | **Homework 8**
due Thursday, March 29

Define, for any real numbers a and b,

$$T_{a,b} = \begin{pmatrix} a+b & b \\ b & a-b \end{pmatrix}.$$

Also define

$$D = \{T_{a,b} \colon a \in \mathbf{Z},\ b \in \mathbf{Z}\} \subseteq M_2(\mathbf{Z}),$$

a subset of $M_2(\mathbf{Z})$ (the set of 2×2 integer matrices).

1. Prove that D is a **subring** of $M_2(\mathbf{Z})$, with the usual matrix addition and multiplication.

2. Prove that D is an integral domain. (**Hint:** Remember that $\sqrt{2}$ is irrational.)

3. Prove that D is an ordered domain with D^+ defined by each of the following two equations:

(a) $D^+ = \{T_{a,b} \colon a + b\sqrt{2} > 0\}$;

(b) $D^+ = \{T_{a,b} \colon a - b\sqrt{2} > 0\}$.

(This is two separate, but very similar, problems. The point is to see that the same integral domain can be ordered in more than one way!)