

Homework 9

due Thursday, April 5

1. Let  $k \in \mathbf{Z}$ , and recall that  $k\mathbf{Z} = \{kn : n \in \mathbf{Z}\}$ . Let

$$R = \{a + b\sqrt{2} : a, b \in \mathbf{Z}\}$$

and let

$$I = \{a + b\sqrt{2} : a, b \in k\mathbf{Z}\}.$$

Prove that  $R$  is a ring and that  $I$  is an ideal in  $R$ .

2. Let  $J$  be an ideal of a commutative ring  $R$ , let  $b \in R$ , and define

$$bJ = \{bj : j \in J\}.$$

Prove that  $bJ$  is an ideal of  $R$ .

3. Let

$$T = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$$

and define  $\theta: M_2(\mathbf{R}) \rightarrow M_2(\mathbf{R})$  by  $\theta(A) = TA$ .

- (a) Prove that  $\theta$  is a homomorphism.
- (b) Find the kernel of  $\theta$ .
- (c) Verify that the kernel of  $\theta$  is an ideal of  $M_2(\mathbf{R})$ .