Math 5370 **GAME THEORY** Spring 2019
Dr. Duval **Homework**

Wednesday, April 10

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Shapley's Theorem

Subsections 12.3.2, 12.3.3

A: Reading questions. Due by 2pm, Sun., 15 Apr.

1. The text claims, right below equation (12.2), that if we set $\psi_i(v) = \phi_i(v, \pi)$ for any fixed π, then the additivity axiom would be satisfied. Verify this claim is true. Then verify the claim, in the third paragraph of the proof of Theorem 12.3.5, that averaging preserves the additivity axiom.
2. Explain how the formula in equation (12.3) matches its verbal description in the paragraph above Remark 12.3.4: "define $\psi_i(v)$ to be the expected value of $\phi_i(v, \pi)$ when π is chosen uniformly at random.
3. Verify the Shapley values $\psi_s(v)$ and $\psi_b(v)$ in Example 12.3.7 (A fish with little intrinsic value).
4. Justify the formulas in the sentence near the end of Example 12.3.8 (Many right gloves), "The number of permutations corresponding to each of these possibilities is $r!, r!, 2(r-1)!, 6(r-1)\cdot(r-2)!$"

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 15 Apr.

Exercise 12.1

Nash bargaining

Section 12.4

A: Reading questions. Due by 2pm, Tue., 16 Apr.

1. Find, and correct, the mistake in Example 12.4.1.
2. Why does Definition 12.4.2 (two-person bargaining problem) need the point $\mathbf{d} = (d_1, d_2)$? (Do not just repeat the definition of $\mathbf{d}$. Explain what would go wrong with the definition if we just left out $\mathbf{d}$.)
3. Rephrase in plain language the last three conditions of the Nash bargaining axioms (Pareto optimality, Symmetry, Independence of Irrelevant Alternatives), and explain why they might be desirable conditions on a bargaining problem.
4. [I'm not sure if this is too hard to ask ...] Draw a picture illustrating the maximization problem being solved by the Nash bargaining solution (Definition 12.4.4). Hint: For the function being maximized, draw several level curves of that function (a level curve of a function is the graph of all points in the domain mapped to a fixed value c, so there is one level curve for each value c). Then draw a picture illustrating Remark 12.4.5. Why can we assume "without loss of generality" that $d_1 = d_2 = 0$?

B: Warmup exercises. For you to present in class. Due by end of class Wed., 17 Apr.

Exercise 12.b.