Math 5370 **ALGEBRAIC STRUCTURES** Spring 2020

Dr. Duval **Homework 2**

due Thursday, February 6

1. Prove that $\{[1],[3],[5],[7]\}$ is a subgroup of the multiplicative group $\mathbf{Z}_8$, but is **not** a **cyclic** subgroup.

2. Prove that $\{a+bi\colon a,b\in\mathbf{Z}; 5a=7b\}$ is a cyclic subgroup of the additive group $\mathbf{C}$.

3. Is the following subset H of $GL(2,\mathbf{R})$ a subgroup of $GL(2,\mathbf{R})$?

$$H=\left\{\begin{bmatrix}1 & b\\ 0 & 1\end{bmatrix} : b\in\mathbf{R}\right\}.$$

Prove your answer is correct.

4. Recall that in class we showed that the set of all six transformations of an equilateral triangle form a group. (If you don't recall that, please contact me.) Name each of the transformations (in any way that makes sense to you), and write down the multiplication table.

Define a **flip** to be any transformation that requires you to change which side of the triangle is showing (in class, this was changing from the asterisk in front to the asterisk in back, or vice versa), and define a **rotation** to be the remaining transformations. Identify which transformations in your table are flips, and which are rotations. [Hint: There should be three of each.]

Prove that the set of all rotations forms a subgroup of the group of transformations. Does the set of all flips form a subgroup? Prove your answer is correct.

Find all the subgroups of order 2 (all subgroups consisting of exactly 2 elements) in the group of transformations, and prove your answer is correct.