Homework 5

due Thursday, February 27

1. Let G be a group, and let $b \in G$. Define $\phi_b\colon G \to G$ by

$$\phi_b(g) = b^{-1}gb.$$

Prove that ϕ_b is an isomorphism from G to itself.

2. Let U be the group of nonsingular upper-triangular 2×2 matrices with real entries,

$$U = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} : a, b, c \in \mathbf{R}, ac \neq 0 \right\},$$

and let T be the subgroup

$$T = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} : x \in \mathbf{R} \right\}.$$

Prove that T is a **normal** subgroup of U.

3. Let D_4 be the group of symmetries of a square, described in Section 4.1, Example 12. (It is described there both geometrically, and in terms of permutations. You may use whichever description you prefer.) Prove that $H = \{e, \alpha^2\}$ is a normal subgroup of D_4.

4. Let D_4 and H be as in problem **3.**. Find the group multiplication table of the quotient group D_4/H.

We know that D_4/H has $8/2 = 4$ elements. From Section 4.4, Example 6, we know that there are only two different groups of order 4 (up to isomorphism): the cyclic group of order 4; and the **Klein four group**. Which of these two groups is D_4/H isomorphic to?