Math 5370 **ALGEBRAIC STRUCTURES** Spring 2020
Dr. Duval **Homework 6**
due Tuesday, March 10

1. Let G be a group, and let H and K be normal subgroups of G. Prove that $H \cap K$ is a normal subgroup of G.

2. Let G be the group of symmetries of a triangle. Recall that we showed in class that the subgroup H of rotations is a normal subgroup of G, and that G/H is isomorphic to $\mathbf{Z}_2$, the cyclic group of order two. Illustrate Theorem 4.25 in this example, by directly describing the epimorphism from G to $\mathbf{Z}_2$, and directly proving (without referring to Theorem 4.25 itself) that it is an epimorphism.

3. Let $Q = \{1, -1, i, -i, j, -j, k, -k\}$ be the quaternion group, and let $\mathbf{Z}_2 = \{e, a\}$ be the cyclic group of order 2 (where e is the identity and $a^2 = e$; we are using this description of $\mathbf{Z}_2$ so that Q and $\mathbf{Z}_2$ do not reuse any names for their respective group elements).

Let $\phi\colon Q \to \mathbf{Z}_2$ satisfy $\phi(i) = \phi(j) = a$. Find $\phi(g)$ for the remaining elements of G so that ϕ is a homomorphism. Prove that ϕ is in fact an epimorphism, and find $\ker \phi$.

Finally, illustrate Theorem 4.28 in this example, by describing $Q/\ker \phi$ and proving it is isomorphic to $\mathbf{Z}_2$, without referring to Theorems 4.27 or 4.28 themselves.