Section 7.3

The Shell Method
The method is called the shell method because it uses cylindrical shells. To find the volume of a solid of a
revolution with the shell method, use one of the following,

\[
\text{Horizontal Axis of revolution (x-axis)} \quad V = 2\pi \int_a^b p(y)h(y)\,dy
\]

\[
\text{Vertical Axis of revolution (y-axis)} \quad V = 2\pi \int_a^b p(x)h(x)\,dx
\]

where \(p\) is the distance between the axis of revolution and the center of the rectangle, \(h\) is the height of
the rectangle, and \(dx\) or \(dy\) becomes the thickness of the cylindrical shell.

**Problem 1.** Use the shell method to set up and evaluate the integral that gives the volume of the solid
generate by revolving the plane region about the \(y\)-axis

a) \(y = x^2 + 4\) with \(x \geq 0\), \(y = 8\), \(x = 0\)

b) \(y = 4 - x^2\), \(y = 0\)
**Problem 2.** Use the shell method to set up and evaluate the integral that gives the volume of the solid generate by revolving the plane region about $x$-axis

a) $x + y^2 = 16$ with $x \geq 0, x = 0$.

b) $y = x^2, x = 0, y = 9$. 
Problem 3. Use the shell method to find the volume of the solid generated by revolving the plane region about the given line. $y = \sqrt{x}$, $y = 0$, $x = 4$, about the line $x = 6$.

Problem 4. Consider the plane region bounded by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > 0$ and $b > 0$. Show that the volume of the ellipsoid formed when this region revolves about the $y$-axis is $\frac{4\pi a^2 b}{3}$.

Homework: Read Section 7.3, do 1-9 (odd), 13 - 27 (odd), 41, 45, 51, 52.