Section 8.7

L'Hôpital’s Rule

Let \( f \) and \( g \) be functions that are differentiable on an open interval \((a, b)\) containing \( c \), except possibly at \( c \) itself. Assume that \( g'(x) \neq 0 \) for all \( x \) in \((a, b)\), except possibly at \( c \) itself. If the limit of \( \frac{f(x)}{g(x)} \) as \( x \) approaches \( c \) produces the indeterminate form \( 0/0 \), then

\[
\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}
\]

provided the limit in the right exists (or is infinite). This result also applies if the limit of \( \frac{f(x)}{g(x)} \) as \( x \) approaches \( c \) produces any one of the indeterminate forms \( \infty/\infty \), \((-\infty)/\infty\), \(\infty/(-\infty)\), or \((-\infty)/(-\infty)\).

The forms \(0/0\), \(\infty/\infty\), \(\infty-\infty\), \(0\cdot\infty\), \(0^0\), \(1^\infty\), and \(\infty^0\) have been identified as indeterminate.

Problem 1. Evaluate the limit, use L'Hôpital’s rule if necessary.

a) \(\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2}\)

b) \(\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1}\)

c) \(\lim_{x \to 0^+} \frac{e^x - x - 1}{x^2}\)


d) \( \lim_{x \to \infty} \frac{x^2}{e^x} \)

e) \( \lim_{x \to \infty} x \tan \frac{1}{x} \)

f) \( \lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x \)
g) \( \lim_{x \to 0^+} (\sin x)^x \)

h) \( \lim_{x \to 1^+} \left( \frac{4}{x - 1} - \frac{5}{\ln x} \right) \)

Homework: Read Section 8.7, do 5-55 (odd).