Section 9.8

Power Series
If \( x \) is a variable, then an infinite series of the form
\[
\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n + \ldots,
\]
is called a power series. More generally, an infinite series of the form
\[
\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2 + \ldots + a_n (x-c)^n + \ldots,
\]
is called a power series at \( c \), where \( c \) is a constant.

An example of power series,
\[
e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots
\]

Convergence of a Power Series
For a power series centered at \( c \), precisely one of the following is true.

a) The series converges only at \( c \).

b) There exists a real number \( R > 0 \) such that the series converges absolutely for \( |x-c| < R \), and diverges for \( |x-c| > R \).

c) The series converges absolutely for all \( x \).

The number \( R \) is called the radius of convergence of the power series.

If the power series converges only at \( c \), the radius of convergence os \( R = 0 \), and if the series converges for all \( x \), the radius of convergence is \( R = \infty \).

The set of all the values for which the power series converges is the interval of convergence of the power series.

Problem 1. Find the radius of convergence of the power series, and the interval of convergence of the power series.

a) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n} \]
b) \[ \sum_{n=0}^{\infty} \frac{(2n)!x^{2n}}{n!} \]

c) \[ \sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!} \]
d) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x - 2)^n}{n2^n} \]

e) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} \]
Problem 2. Let \( f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \ldots \).

Find \( f'(x) \) and \( \int f(x) \, dx \).

Homework: Read Section 9.8, do 3-9 (odd), 11-29 (odd).