Section 5.3

Future Value of a Sinking Fund
A **sinking fund** is an account earning compound interest into which you make periodic deposits. Suppose that the account has an annual rate of \( r \) compounded \( m \) times per year, so that \( i = \frac{r}{m} \) is the interest rate per compounding period. If you make a payment of \( PMT \) at the end of each period, then the future value after \( t \) years, or \( n = m \cdot t \) periods, will be

\[
FV = PMT \cdot \frac{(1 + \frac{r}{m})^{m \cdot t} - 1}{\frac{r}{m}} \quad \text{or} \quad FV = PMT \cdot \frac{(1 + i)^n - 1}{i}
\]

Payment Formula for a Sinking Fund
Suppose that an account has an annual rate of \( r \) compounded \( m \) times per year, so that \( i = \frac{r}{m} \) is the interest rate per compounding period. If you want to accumulate a total of \( FV \) in the account after \( t \) years, or \( n = m \cdot t \) periods, by making payments of \( PMT \) at the end of each period, then each payment must be

\[
PMT = FV \cdot \frac{i}{(1 + i)^n - 1}
\]

Present Value of an Annuity
An **annuity** is an account earning compound interest from which periodic withdrawals are made. Suppose that the account has an annual rate of \( r \) compounded \( m \) times per year, so that \( i = \frac{r}{m} \) is the interest rate per compounding period. Suppose also that the account starts with a balance of \( PV \). If you receive a payment of \( PMT \) at the end of each compounding period, and the account is down to $0 after \( t \) years, or \( n = m \cdot t \) periods, then

\[
PV = PMT \cdot \frac{1 - (1 + i)^{-n}}{i}
\]

Payment Formula for an Ordinary Annuity
Suppose that an account has an annual rate of \( r \) compounded \( m \) times per year, so that \( i = \frac{r}{m} \) is the interest rate per compounding period. Suppose also that the account starts with a balance of \( PV \). If you want to receive a payment of \( PMT \) at the end of each compounding period, and the account is down to $0 after \( t \) years, or \( n = m \cdot t \) periods, then

\[
PMT = PV \cdot \frac{i}{1 - (1 + i)^{-n}}
\]
Problem 1. Find the amount accumulated in the sinking funds in the following exercises 1–2. (Assume end-of-period deposits and compounding at the same intervals as deposits.)

- $100 deposited monthly for 10 years at 5% per year
- $150 deposited monthly for 20 years at 3% per year

Problem 2. Find the periodic payments necessary to accumulate the amounts given in the following exercises in a sinking fund. (Assume end-of-period deposits and compounding at the same intervals as deposits.)

- $10,000 in a fund paying 5% per year, with monthly payments for 5 years
- $20,000 in a fund paying 3% per year, with monthly payments for 10 years
- $75,000 in a fund paying 6% per year, with quarterly payments for 20 years

Problem 3. Find the present value of the annuity necessary to fund the withdrawals given in the following exercises. (Assume end-of-period withdrawals and compounding at the same intervals as withdrawals.)

- $500 per month for 20 years, if the annuity earns 3% per year
- $1000 per month for 15 years, if the annuity earns 5% per year
- $1500 per quarter for 20 years, if the annuity earns 6% per year

Problem 4. Find the periodic withdrawals for the annuities given in the following exercises. (Assume end-of-period withdrawals and compounding at the same intervals as withdrawals.)

- $100,000 at 3%, paid out monthly for 20 years
- $150,000 at 5%, paid out monthly for 15 years
- $75,000 at 4%, paid out quarterly for 20 years
- $200,000 at 6%, paid out quarterly for 15 years