Using Error-Eliciting Problems To Help Students Overcome Their Impulsive Tendency

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Outline of Presentation

- Why Focus on Errors?
- Error-Eliciting Problems
- Impulsive Tendency
- Let’s Vote
- Practical Suggestions
Why Focus on Errors?

- Errors can stimulate inquiry, discussion, and reflection (Borasi, 1994; Kramarski & Zoldan, 2008)

“Mistakes are seen not as dead ends but rather as potential avenues for learning.” (NCTM, 2000, p144)
Why Focus on Errors?

- Errors can stimulate inquiry, discussion, and reflection (Borasi, 1994; Kramarski & Zoldan, 2008)
- Swan (1983) found that an error-remediation approach was more effective than reteaching
- Lee (1995) found that students showed greater improvement when there is a conceptual focus in the error-remediation approach
- Borasi (1994) found that error activities offered various learning opportunities including:
  - experiencing doubts and conflicts
  - pursuing mathematical explorations
  - reflecting on the nature of mathematics
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What is an Error-Eliciting Problem?

A math task that has the potential to reveal common errors caused by

- A misconception
- A misapplication of an idea
- An overgeneralization of an idea
Why Use Error-Eliciting Problems?

- To make an error explicit
- To resolve misconception
- To allow students to realize their impulsive tendency
- To foster certain mathematical ways of thinking (or habits of mind)
To Elicit a Misconception

45% x 2% = ______

A. 0.009
B. 0.09
C. 0.9
D. 9
E. 9000
To Elicit a Misconception

What misconception does this problem elicit?

What way of thinking can this problem foster?

45% x 2% = \[90\%\] = 0.9

A. 0.009 41%  \(N = 32\)
B. 0.09 16%
C. 0.9 34%
D. 9 9%
E. 9000 0%

45% x 2% = 45 \times 0.01 \times 2 \times 0.01
= 90 \times 0.01^2

3 \text{ cm} \times 5 \text{ cm} = 15 \text{ cm}^2
Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20th minute?

A. Approximately 3000 meters
B. Approximately 50 meters/min
C. Approximately 80 meters/min
D. Approximately 150 meters/min
Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20th minute?

(a) Approximately 3000 meters
(b) Approximately 50 meters/min
(c) Approximately 80 meters/min
(d) Approximately 150 meters/min

Answer: C

307 Pre-service EC-4 Teachers
Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20th minute?

- (a) Approximately 3000 meters 28%
- (b) Approximately 50 meters/min 18%
- (c) Approximately 80 meters/min 52%
- (d) Approximately 150 meters/min 2%

Answer: A B C D

307 Pre-service EC-4 Teachers
Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20th minute?

**Standard Formula**

\[
\text{Speed} = \frac{\text{Distance}}{\text{Time}}
\]

\[
\mathbf{s} = \frac{d}{t}
\]
Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20th minute?

To Elicit a Misapplication of a Formula

Standard Formula

\[ s = \frac{d}{t} \]

Correct Interpretation

\[ s = \frac{\Delta d}{\Delta t} \]
The ratio of the volume of a small glass to the volume of a large glass is 3:5. If it takes 15 small glasses to fill the container, how many large glasses does it take to fill the container?

A. 9 glasses  53%
B. 13 glasses  9%
C. 17 glasses  4%
D. 25 glasses  24%
E. None of the above  10%

N = 138 (Written Pre-Test)
Direct-Proportional Item
The ratio of the amount of soda in the can to the amount of soda in the bottle is 4:3. There are 12 fluid ounces of soda in the can, how many fluid ounces of soda are in the bottle?

(a) 8 fluid ounces
(b) 9 fluid ounces
(c) 15 fluid ounces
(d) 16 fluid ounces
(e) None of the above

138 students

Pretest | Posttest
---|---
3% | 6%
64% | 78%
6% | 3%
27% | 11%
1% | 2%

Inverse-Proportional Item
The ratio of the volume of a small glass to the volume of a large glass is 3:5. If it takes 15 small glasses to fill the container, how many large glasses does it take to fill the container?

(a) 9 glasses
(b) 13 glasses
(c) 17 glasses
(d) 25 glasses
(e) None of the above

Pretest | Posttest
---|---
53% | 42%
9% | 13%
4% | 2%
24% | 40%
10% | 2%
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Impulsive Tendency

- **Cognitive Style** (Kagan et al., 1964)
  - An *impulsive* person performs tasks rapidly, but usually makes more mistakes.
  - A *reflective* person is slower but more accurate.

- **Problem-solving Disposition** (Lim, Morera, & Tchoshanov, 2009)
  - *Impulsive disposition* refers to one’s tendency to spontaneously proceed with an action that comes to mind.
  - *Analytic disposition* refers to one’s tendency to analyze a problem situation.
Impulsive Tendency

- **Cognitive Style** (Kagan et al., 1964)
  - A personality trait
  - Stable across situation and across time
  - A dichotomy: impulsive versus reflective
  - Characterized by fast-inaccurate responses

- **Problem-solving Disposition** (Lim, Morera, & Tchoshanov, 2009)
  - A tendency to act
  - Context-dependent and modifiable
  - A continuum along impulsive-analytic dimension
  - Inferred from errors during problem solving
How Can We Account For It?

- Human Nature

**Dual Process Theories** (Evans, 2006; Reber, 1993; Sloman, 1996; Stanovich & West, 2000)

**Two Distinct System of Cognitive Reasoning**

- **System 1**
  - Associative system
  - Low-level conditioning process
  - Rapid, Automated
  - Implicit, Unconscious
  - Beliefs-based

- **System 2**
  - Rule-based system
  - Higher-order cognitive processes
  - Slow, Effortful
  - Explicit, Conscious
  - Logic-based
How Can We Account For It?

- Human Nature
  - Dual Process Theories
  - Intuitive Rules (Tirosh & Stavy, 1998)
    - More A More B
      (e.g. the heavier the object, the faster it falls)
    - Same A Same B
      (e.g. same side length, same angle)

  “... accounts for many of the observed incorrect responses to science and mathematics tasks” (p. 85)
How Can We Account For It?

- Human Nature
  - Dual Process Theories
  - Intuitive Rules

- School Effect (i.e. Nurture)

  “The tradition has been to regard ‘mathematics’ as a set of rules for writing symbols on paper, and to regard the ‘teaching’ of mathematics as merely a matter of ‘telling’ students what to write and where to write it, together with supervising some considerable amount of drill and practice.”

  (David, 1989, p. 159)
How Can We Account For It?

- **Human Nature**
  - Dual Process Theories
  - Intuitive Rules

- **School Effect (i.e. Nurture)**
  - Compartamentalization of school mathematics
  - Performance-oriented curriculum
    - acronyms (e.g., FOIL)
    - schematic tools (e.g., ratio box to find the missing value from three given values)
    - key words (e.g., altogether means add)
    - associations (e.g., speed problem, work-rate problem)
How Can We Account For It?

- **Human Nature**
  - Dual Process Theories
  - Intuitive Rules

- **School Effect (i.e. Nurture)**
  - Compartmentalization of school mathematics
  - Performance-oriented curriculum
  - Clear-and-easy-to-remember instruction
  - Initiate-Respond-Evaluate (IRE) interaction
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- Practical Suggestions
Jose is offered a raise if he can increase his weekly productivity by 12%. If Jose works a four-day week, how much does he need to increase his productivity each day?

A. By 3% each day  
B. By 4% each day  
C. By 12% each day  
D. By 36% each day  
E. By 48% each day

59%  
0%  
38%  
0%  
3%
Jose is offered a raise if he can increase his weekly productivity by 12%. If Jose works a four-day week, how much does he need to increase his productivity each day?

12% of 400 = 48

12% for each day means

4 x 12% of 100 = 4 x 12
Jose is offered a raise if he can increase his weekly productivity by 12%. If Jose works a four-day week, how much does he need to increase his productivity each day?

**Common Misconception**
12% for 4 days means 3% for each day

**Correct Conception**
12% for 4 days means 12% for each day

400 = 4 x 100
12% of 400 = 48
3% of 100 = 3

= 4 x 3% of 100

≠ 4 x 3% of 100

4 x ___% of 100

3

12
An object is moving along a straight line between point P and point Q. The distance of the object from point P is denoted by \( x \) meters. The graph shows how the object’s velocity, \( \frac{dx}{dt} \), varies with time, \( t \) seconds.

In which intervals of time is the value of \( x \) increasing?

A. Between 0 second and 9 seconds
B. Between 6 seconds and 14 seconds
C. Between 9 seconds and 14 seconds
D. Between 9 seconds and 20 seconds

\( N = 26 \text{ MAT} + 6 \text{ undergrads} \)
An object is moving along a straight line between point P and point Q. The distance of the object from point P is denoted by $x$ meters. The graph shows how the object’s velocity, $dx/dt$, varies with time, $t$ seconds.

In which intervals of time is the value of $x$ increasing?

Potential Misconception
$x$ is increasing if the graph is increasing

Correct Conception
$x$ is increasing when $dx/dt$ is positive
Practical Suggestions

Creating and Using Error-Eliciting Problems

- Note student common misconceptions
- Construct problems that elicit those misconceptions
- Specify the learning objective(s) for each problem
- Think about implementation issues
  - When?
  - How?
Practical Suggestions

Implementing Error-Eliciting Problems

- Consider classroom voting (clickers or voting cards)
Benefits of Using Clickers

- Requires students to participate actively
- Provides immediate feedback
- Facilitates class discussion/debate
- Creates a fun atmosphere

(Cline, Zullo, & Parker, 2006)
Benefits of Using Clickers

 Requires students to participate actively
 Provides immediate feedback
 Facilitates class discussion/debate
 Creates a fun atmosphere
 Provides a safe environment

“Classroom voting is fun. Students like the act of participating and enjoy the ‘game show’ atmosphere of the process. ... When students have fun in class ... their minds will be awake and involved, and they will be ready to learn.”

(Cline, 2006, p. 101)
Implementing Error-Eliciting Problems

Consider classroom voting

1. **Students work individually and then vote**
2. **Students discuss among themselves and then re-vote**
3. **Students explain their reasons for their answer choices**
4. **Teacher orchestrates whole-class discussion**
   - Make sure each explanation is understood by all students
   - Let students challenge each others ideas
   - Press students for justification
Implementing Error-Eliciting Problems

- Consider classroom voting
- Consider adapting Smith & Stein’s 5 Practices for Orchestrating Classroom Discussion

1. Anticipating
2. Monitoring
3. Selecting
4. Sequencing
5. Connecting
Practical Suggestions

Implementing Error-Eliciting Problems

- Consider classroom voting
- Consider adapting Smith & Stein’s *5 Practices for Orchestrating Classroom Discussion*
- Have students reflect on their mistakes and write a summary of what they have learned
Thank You