Using Error-Eliciting Problems To Help Students Overcome Their Impulsive Tendency

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Outline of Presentation

- Why Focus on Errors?
- Error-Eliciting Problems
- Impulsive Tendency
- Let’s Vote
- Practical Suggestions

Why Focus on Errors?

- Errors can stimulate inquiry, discussion, and reflection (Borasi, 1994; Kramarski & Zoldan, 2008)

"Mistakes are seen not as dead ends but rather as potential avenues for learning."
(NCTM, 2000, p144)

Why Focus on Errors?

- Errors can stimulate inquiry, discussion, and reflection (Borasi, 1994; Kramarski & Zoldan, 2008)
- Swan (1983) found that an error-remediation approach was more effective than reteaching
- Lee (1995) found that students showed greater improvement when there is a conceptual focus in the error-remediation approach
- Borasi (1994) found that error activities offered various learning opportunities including:
  - experiencing doubts and conflicts
  - pursuing mathematical explorations
  - reflecting on the nature of mathematics

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What is an Error-Eliciting Problem?

A math task that has the potential to reveal common errors caused by

- A misconception
- A misapplication of an idea
- An overgeneralization of an idea

Why Use Error-Eliciting Problems?

- To make an error explicit
- To resolve misconception
- To allow students to realize their impulsive tendency
- To foster certain mathematical ways of thinking (or habits of mind)

To Elicit a Misconception

45% x 2% = ______

A. 0.009
B. 0.09
C. 0.9
D. 9
E. 9000

To Elicit a Misconception

What misconception does this problem elicit?

What way of thinking can this problem foster?

Attending to Meaning of Symbols

45% x 2% = 90% = 0.9

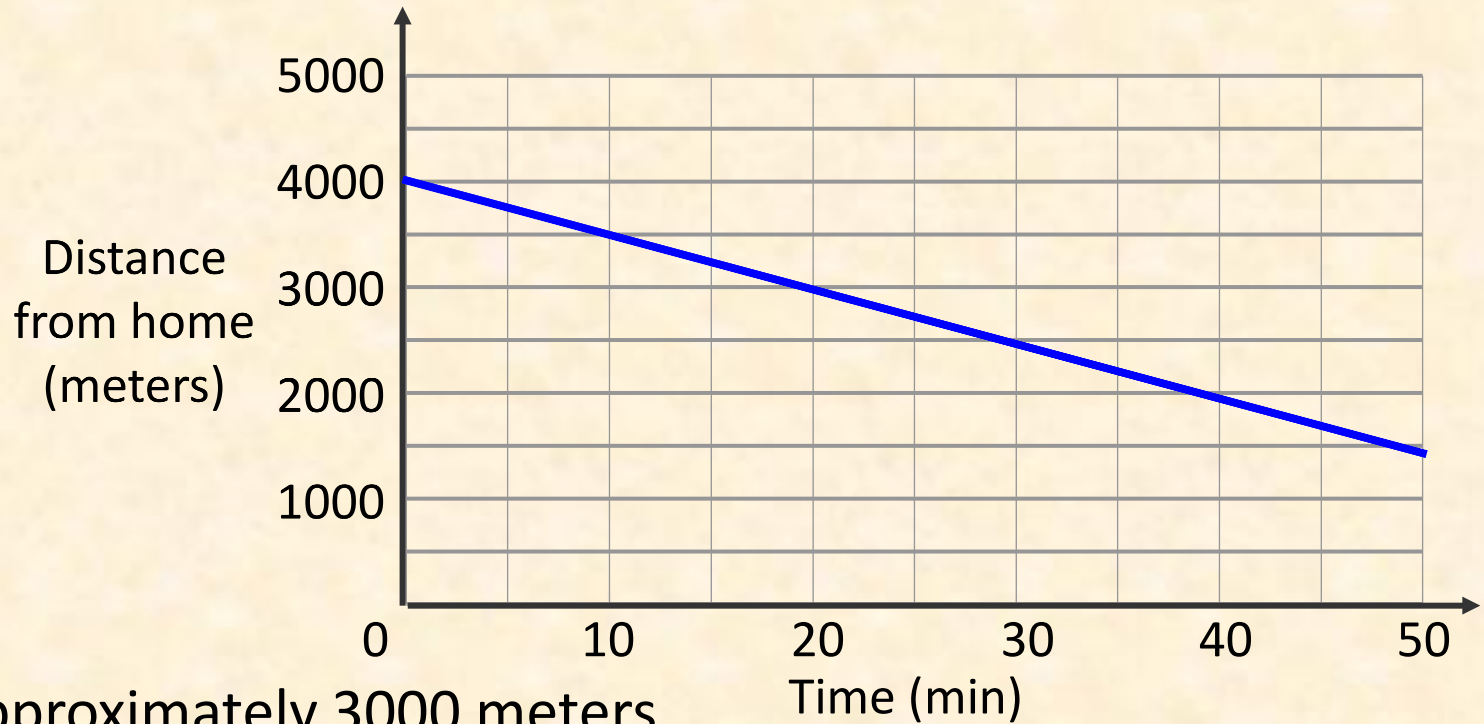
A. 0.009	41%	N = 32
B. 0.09	16%	
C. 0.9	34%	
D. 9	9%	
E. 9000	0%	

$$45\% \times 2\% = 45 \times 0.01 \times 2 \times 0.01$$
$$= 90 \times 0.01^2$$

$$3 \text{ cm} \times 5 \text{ cm} = 15 \text{ cm}^2$$

To Elicit a Misapplication of a Formula

Gina is traveling home from her friend's house. The graph represents a portion of Gina's journey. What is Gina's speed at the 20th minute?



A. Approximately 3000 meters
B. Approximately 50 meters/min
C. Approximately 80 meters/min
D. Approximately 150 meters/min

To Elicit a Misapplication of a Formula

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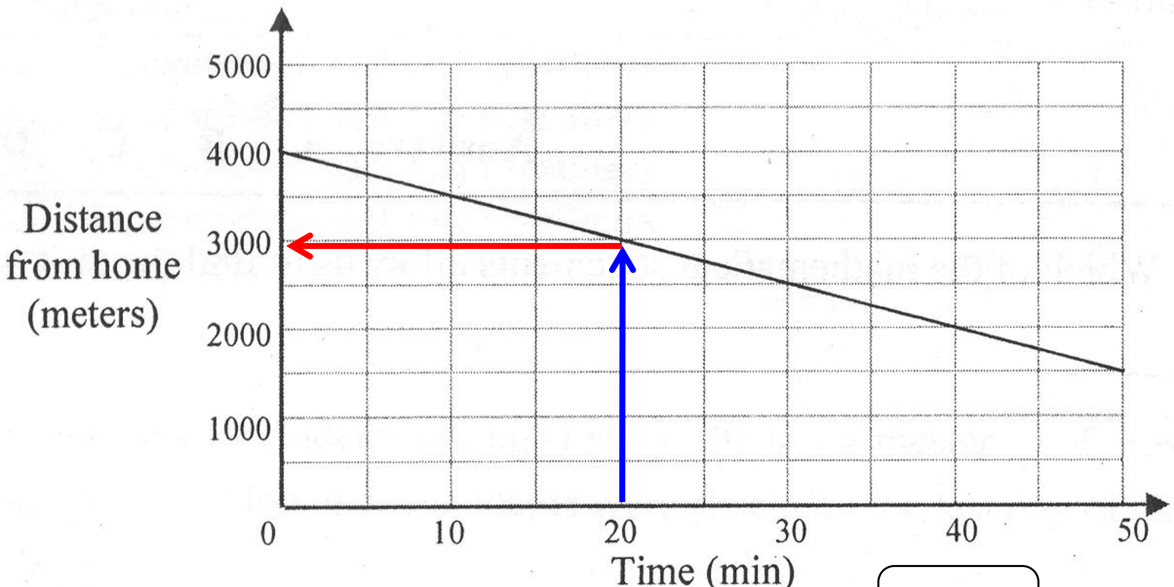

Speed = $\frac{d}{t}$ = $\frac{3000}{20}$ = 150

(a) Approximately 3000 meters — 28%
(b) Approximately 50 meters/min
(c) Approximately 80 meters/min
(d) Approximately 150 meters/min — 52%

Answer: A B C (D)

307 Pre-service EC-4 Teachers

To Elicit a Misapplication of a Formula

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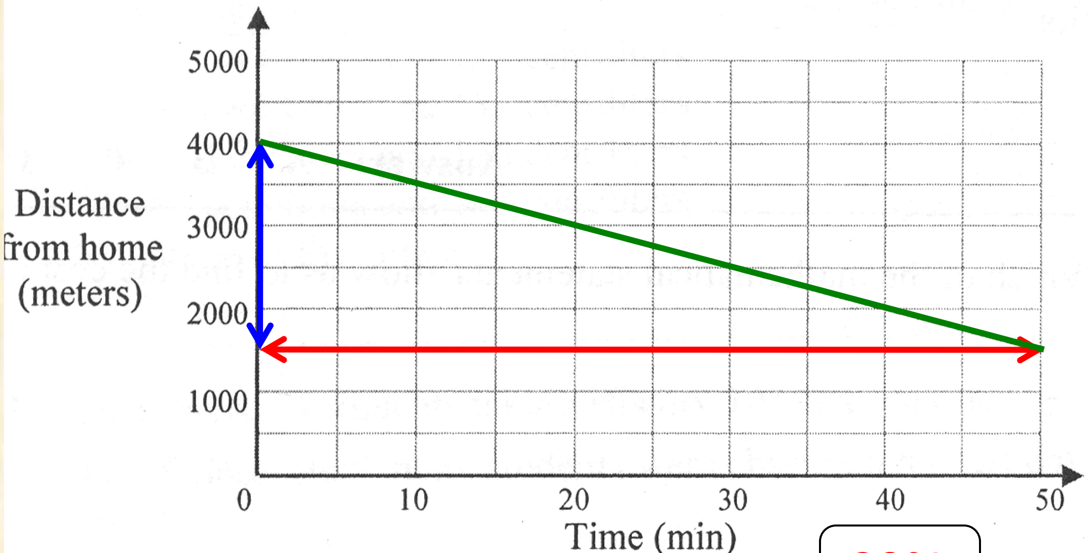

(a) Approximately 3000 meters — 28%
(b) Approximately 50 meters/min — 18%
(c) Approximately 80 meters/min — 2%
(d) Approximately 150 meters/min — 52%

Answer: A B C D

307 Pre-service EC-4 Teachers

To Elicit a Misapplication of a Formula

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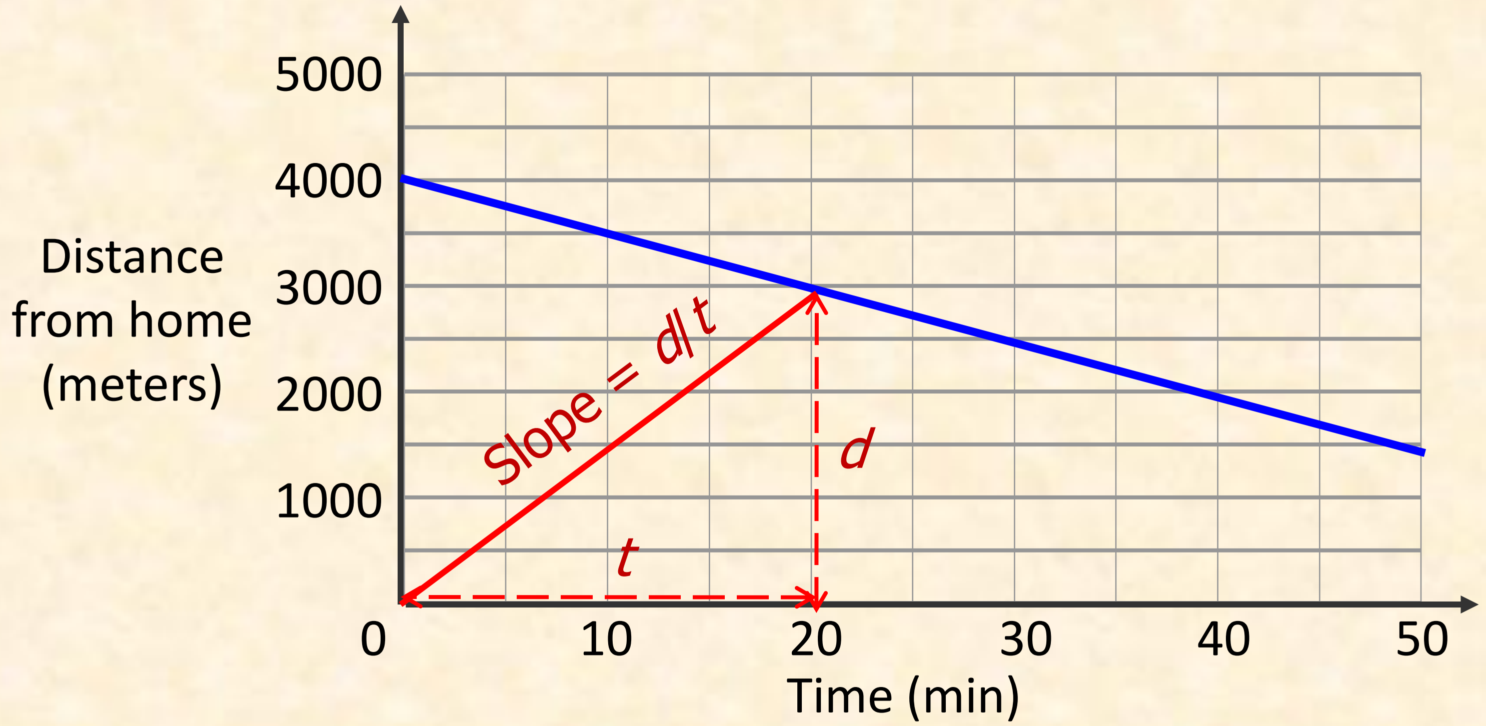

Standard Formula

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$s = \frac{d}{t}$$

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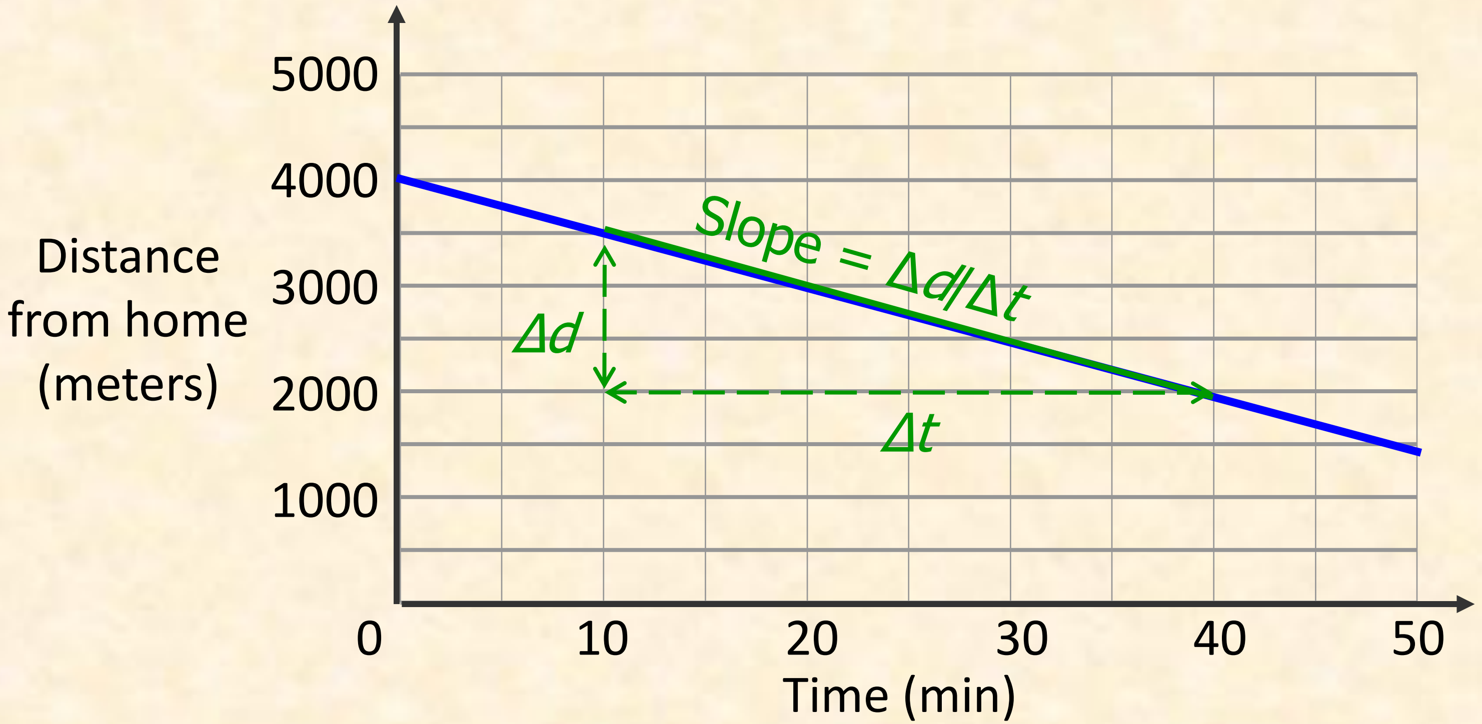


Standard Formula

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$s = \frac{d}{t}$$

Correct Interpretation

$$\text{Speed} = \frac{\text{Distance traveled}}{\text{Time taken}}$$

$$s = \frac{\Delta d}{\Delta t}$$

To Elicit Students' Overgeneralization of a Concept

Which concept are students overgeneralizing? Proportionality

What undesirable habits of mind are students displaying?

Impulsive Disposition

The ratio of the volume of a small glass to the volume of a large glass is 3:5. If it takes 15 small glasses to fill the container, how many large glasses does it take to fill the container?

N = 138 (Written Pre-Test)

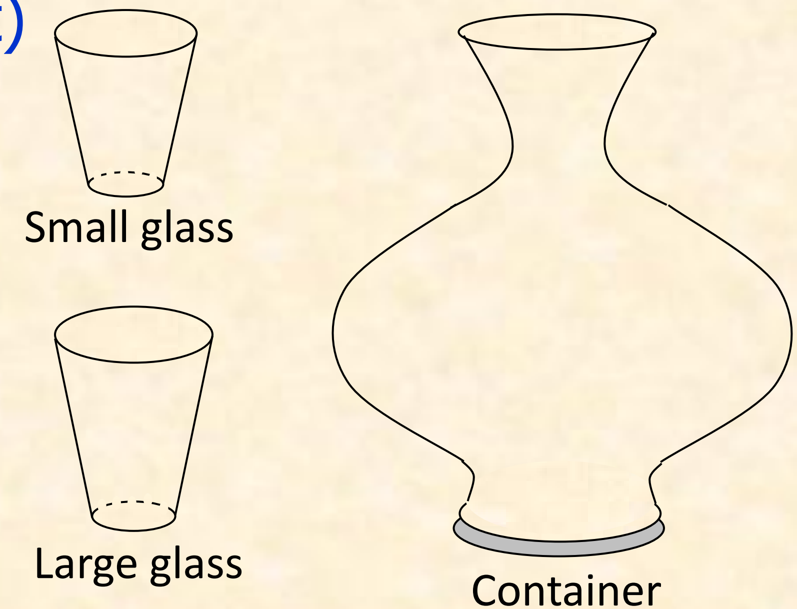
A. 9 glasses 53%

B. 13 glasses 9%

C. 17 glasses 4%

D. 25 glasses 24%

E. None of the above 10%



To Elicit Students' Overgeneralization of a Concept

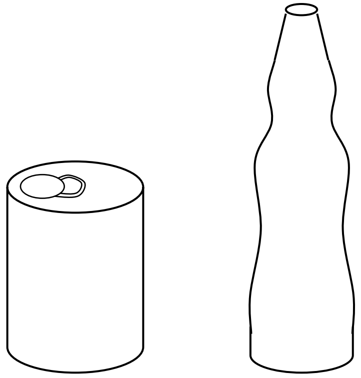
Students' Tendency to Apply Recently Learned Ideas

138 students

Direct-Proportional Item

The ratio of the amount of soda in the can to the amount of soda in the bottle is 4:3. There are 12 fluid ounces of soda in the can, how many fluid ounces of soda are in the bottle?

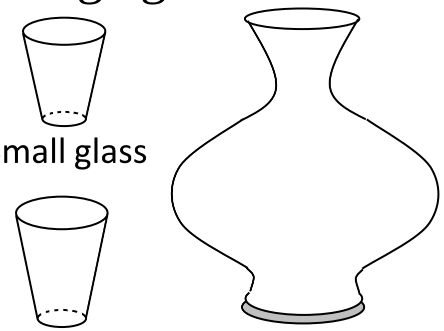
		Pretest	Posttest
(a)	8 fluid ounces	3%	6%
(b)	9 fluid ounces	64%	78%
(c)	15 fluid ounces	6%	3%
(d)	16 fluid ounces	27%	11%
(e)	None of the above	1%	2%



Inverse-Proportional Item

The ratio of the volume of a small glass to the volume of a large glass is 3:5. If it takes 15 small glasses to fill the container, how many large glasses does it take to fill the container?

		Pretest	Postest
(a)	9 glasses	53%	42%
(b)	13glasses	9%	13%
(c)	17glasses	4%	2%
(d)	25 glasses	24%	40%
(e)	None of the above	10%	2%



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Impulsive Tendency

- Cognitive Style (Kagan et al., 1964)
  - An *impulsive* person performs tasks rapidly, but usually makes more mistakes.
  - A *reflective* person is slower but more accurate.

- Problem-solving Disposition (Lim, Morera, & Tchoshanov, 2009)
  - *Impulsive disposition* refers to one's tendency to spontaneously proceed with an action that comes to mind.
  - *Analytic disposition* refers to one's tendency to analyze a problem situation.

Impulsive Tendency

- Cognitive Style (Kagan et al., 1964)
  - A personality trait
  - Stable across situation and across time
  - A dichotomy: impulsive versus reflective
  - Characterized by fast-inaccurate responses
- Problem-solving Disposition (Lim, Morera, & Tchoshanov, 2009)
  - A tendency to act
  - Context-dependent and modifiable
  - A continuum along impulsive-analytic dimension
  - Inferred from errors during problem solving

How Can We Account For It?

- Human Nature

  Dual Process Theories (Evans, 2006; Reber, 1993; Sloman, 1996; Stanovich & West, 2000)

Two Distinct System of Cognitive Reasoning

• System 1	System 2
• Associative system	Rule-based system
• Low-level conditioning process	Higher-order cognitive processes
• Rapid, Automated	Slow, Effortful
• Implicit, Unconscious	Explicit, Conscious
• Beliefs-based	Logic-based

How Can We Account For It?

- Human Nature

  Dual Process Theories

  Intuitive Rules (Tirosh & Stavy, 1998)

  - More A More B
    (e.g. the heavier the object, the faster it falls)
  - Same A Same B
    (e.g. same side length, same angle)

  "… accounts for many of the observed incorrect responses to science and mathematics tasks" (p. 85)

How Can We Account For It?

- Human Nature

  Dual Process Theories

  Intuitive Rules

- School Effect (i.e. Nurture)

  "The tradition has been to regard 'mathematics' as a set of rules for writing symbols on paper, and to regard the 'teaching' of mathematics as merely a matter of 'telling' students what to write and where to write it, together with supervising some considerable amount of drill and practice."
  (David, 1989, p. 159)

How Can We Account For It?

- Human Nature
  - Dual Process Theories
  - Intuitive Rules
- School Effect (i.e. Nurture)
  - Compartmentalization of school mathematics
  - Performance-oriented curriculum
    - acronyms (e.g., FOIL)
    - schematic tools (e.g., ratio box to find the missing value from three given values)
    - key words (e.g., altogether means add)
    - associations (e.g., speed problem, work-rate problem)

How Can We Account For It?

- Human Nature
  - Dual Process Theories
  - Intuitive Rules
- School Effect (i.e. Nurture)
  - Compartmentalization of school mathematics
  - Performance-oriented curriculum
  - Clear-and-easy-to-remember instruction
  - Initiate-Respond-Evaluate (IRE) interaction

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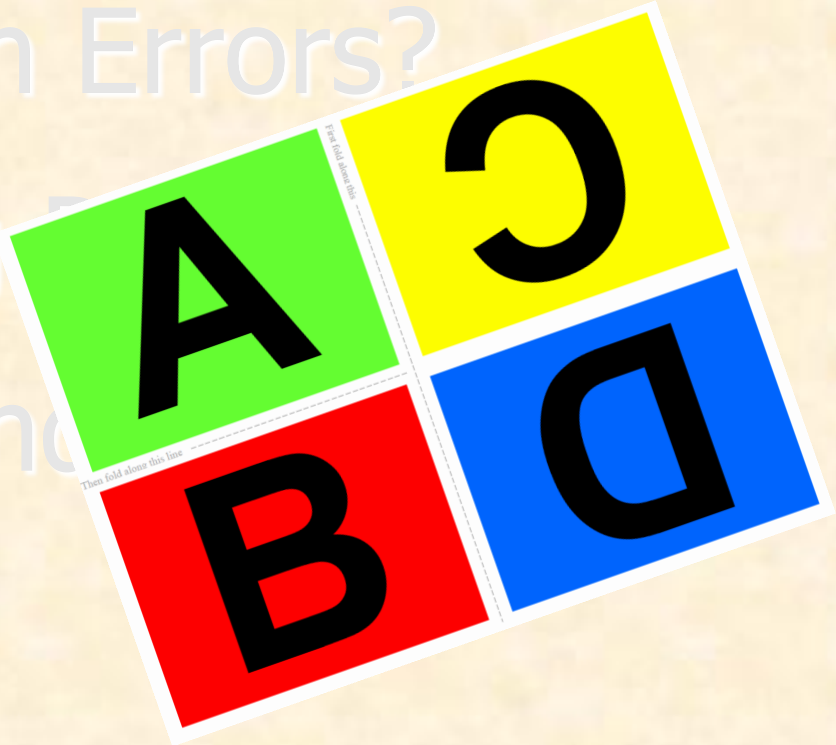


More Problems: #1

Jose is offered a raise if he can increase his weekly productivity by 12%. If Jose works a four-day week, how much does he need to increase his productivity each day?

A.	By 3% each day	59%
B.	By 4% each day	0%
C.	By 12% each day	38%
D.	By 36% each day	0%
E.	By 48% each day	3%

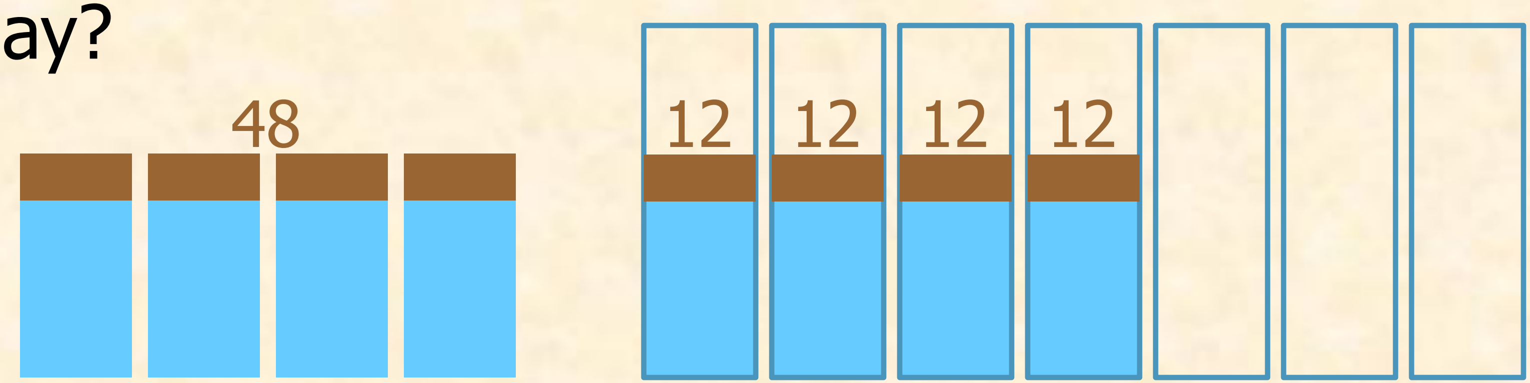
N = 37
27 in-service
10 pre-service

Correct Conception

12% for 4 days means

12% for each day

Jose is offered a raise if he can increase his weekly productivity by 12%. If Jose works a four-day week, how much does he need to increase his productivity each day?



$$400 = 4 \times 100$$

$$\underbrace{12\% \text{ of } 400}_{48} = 4 \times \underbrace{\underline{\mathbf{12}}\ \% \text{ of } 100}_{12}$$

Common Misconception

12% for 4 days means

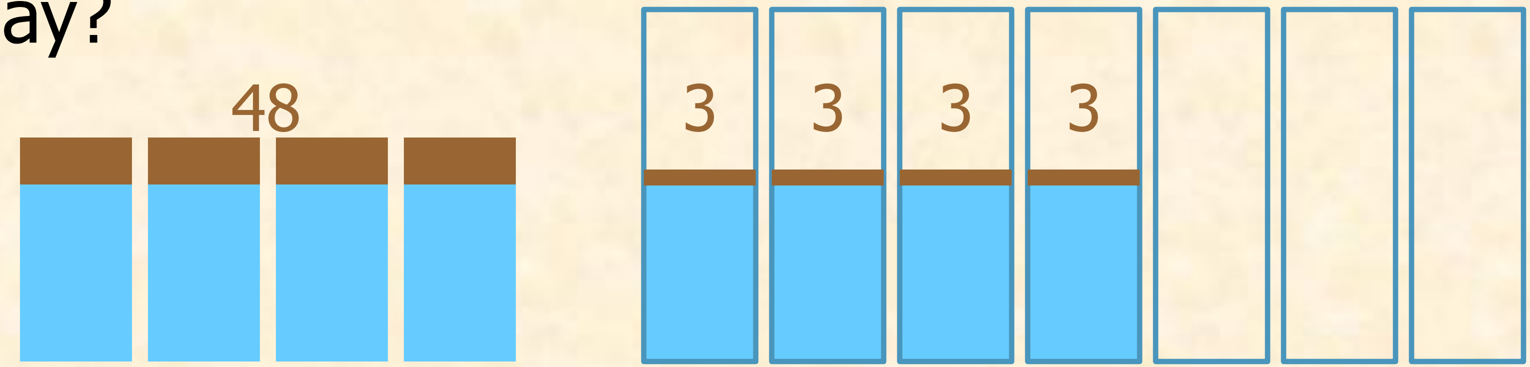
3% for each day

Correct Conception

12% for 4 days means

12% for each day

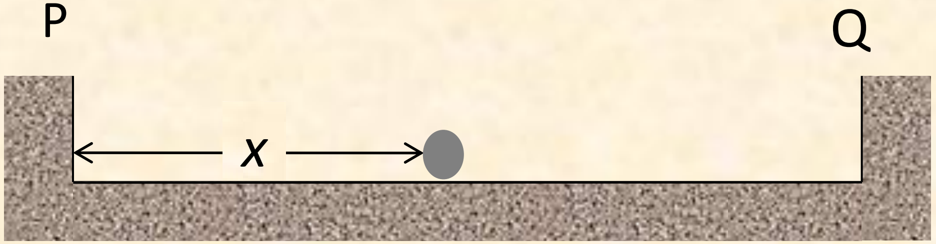
Jose is offered a raise if he can increase his weekly productivity by 12%. If Jose works a four-day week, how much does he need to increase his productivity each day?

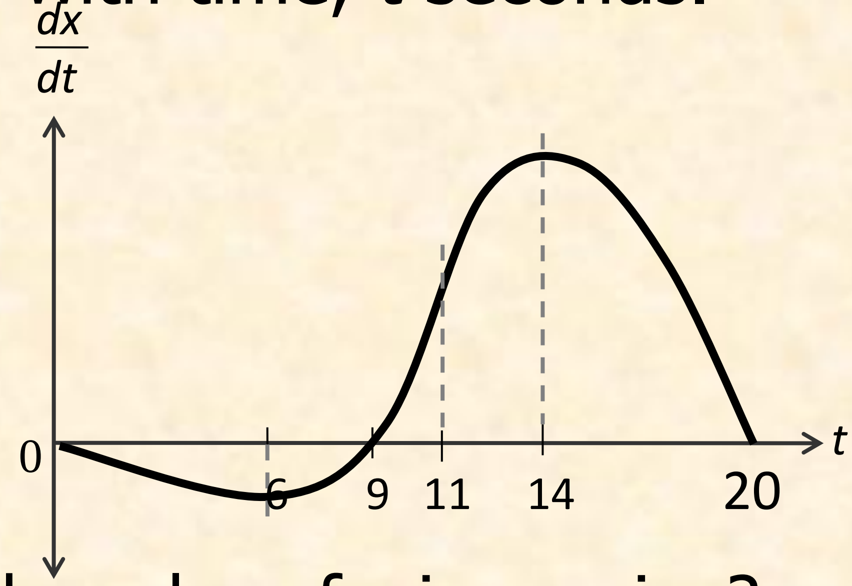


$400 = 4 \times 100$

$\underbrace{12\% \text{ of } 400}_{48} \neq \underbrace{4 \times \underline{\ \mathbf{3}\ }\% \text{ of } 100}_{12}$

An object is moving along a straight line between point P and point Q. The distance of the object from point P is denoted by *x* meters. The graph shows how the object's velocity, *dx/dt*, varies with time, *t* seconds.





N = 26 MAT + 6 undergrads

In which intervals of time is the value of *x* increasing?

A. Between 0 second and 9 seconds 6%
B. Between 6 seconds and 14 seconds 41%
C. Between 9 seconds and 14 seconds 47%
D. Between 9 seconds and 20 seconds 6%

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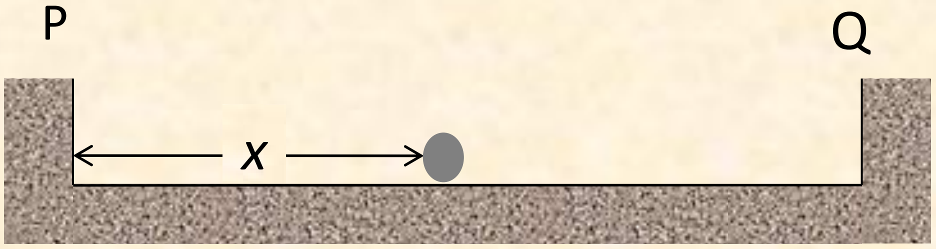
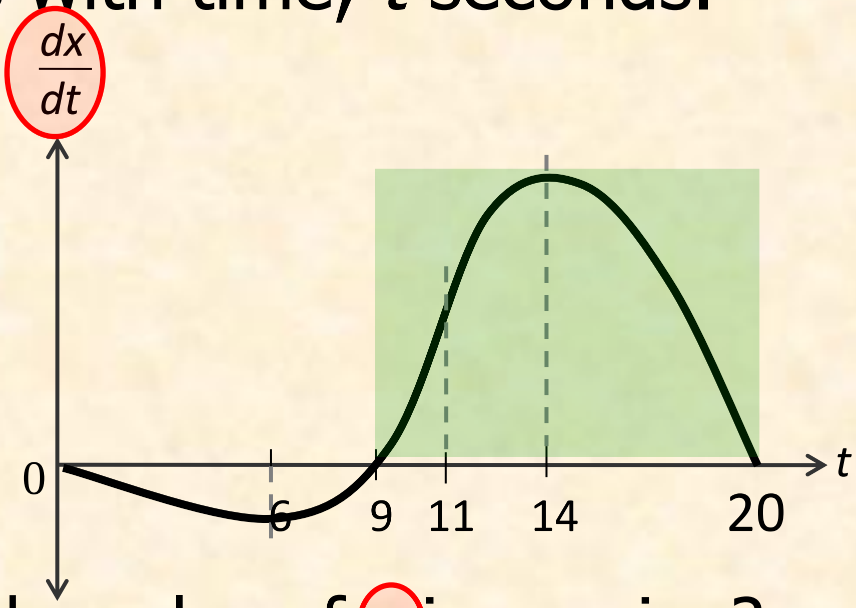




In which intervals of time is the value of *x* increasing?

Potential Misconception

x is increasing if
the graph is increasing

Correct Conception

x is increasing when
dx/ *dt* is positive

Practical Suggestions

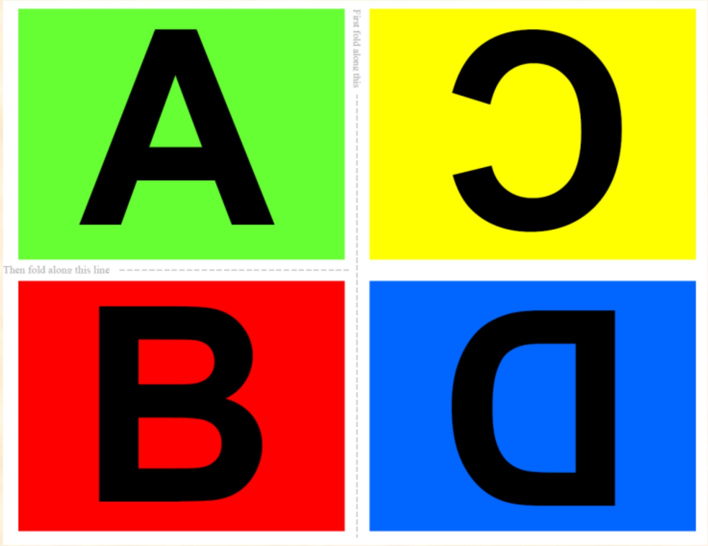
Creating and Using Error-Eliciting Problems

- Note student common misconceptions
- Construct problems that elicit those misconceptions
- Specify the learning objective(s) for each problem
- Think about implementation issues
  - When?
  - How?

Practical Suggestions

Implementing Error-Eliciting Problems

- Consider classroom voting
  (clickers or voting cards)



Benefits of Using Clickers

- Requires students to participate actively
- Provides immediate feedback
- Facilitates class discussion/debate
- Creates a fun atmosphere

(Cline, Zullo, & Parker, 2006)

Benefits of Using Clickers

- Requires students to participate actively
- Provides immediate feedback
- Facilitates class discussion/debate
- Creates a fun atmosphere
- Provides a safe environment

"Classroom voting is fun. Students like the act of participating and enjoy the 'game show' atmosphere of the process. … When students have fun in class … their minds will be awake and involved, and they will be ready to learn."

(Cline, 2006, p. 101)

Implementing Error-Eliciting Problems

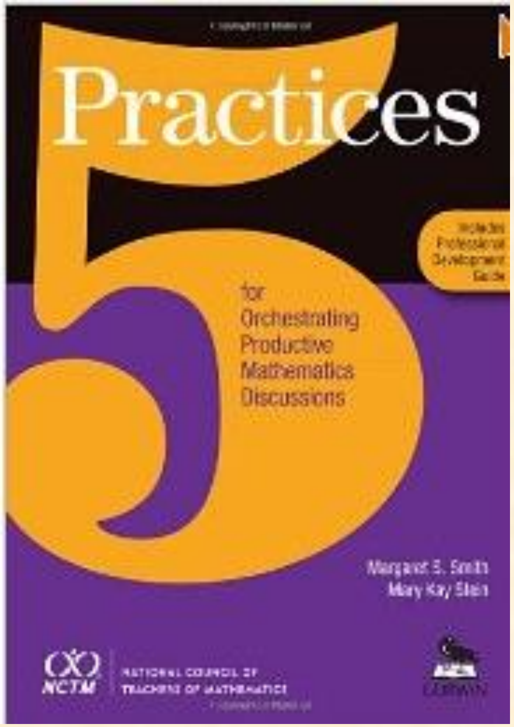
- Consider classroom voting
  1. Students work individually and then vote
  2. Students discuss among themselves and then re-vote
  3. Students explain their reasons for their answer choices
  4. Teacher orchestrates whole-class discussion
     - Make sure each explanation is understood by all students
     - Let students challenge each others ideas
     - Press students for justification

Practical Suggestions

Implementing Error-Eliciting Problems

- Consider classroom voting
- Consider adapting Smith & Stein's *5 Practices for Orchestrating Classroom Discussion*

1. Anticipating
2. Monitoring
3. Selecting
4. Sequencing
5. Connecting



Practical Suggestions

Implementing Error-Eliciting Problems

- Consider classroom voting
- Consider adapting Smith & Stein’s *5 Practices for Orchestrating Classroom Discussion*
- Have students reflect on their mistakes and write a summary of what they have learned

Thank You