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Helping Students Overcome Their Tendency to Apply Procedures without Thinking

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Presentation Outline

1. Introduction
2. Two Types of Dispositions
3. Pedagogical Suggestions
4. Conclusion

Students' Application of Procedures without Thinking: Evidence #1

$$(b-1)(b+4)=0$$

$$b^2+4b-b-4=0$$

$$b^2+3b-4=0$$

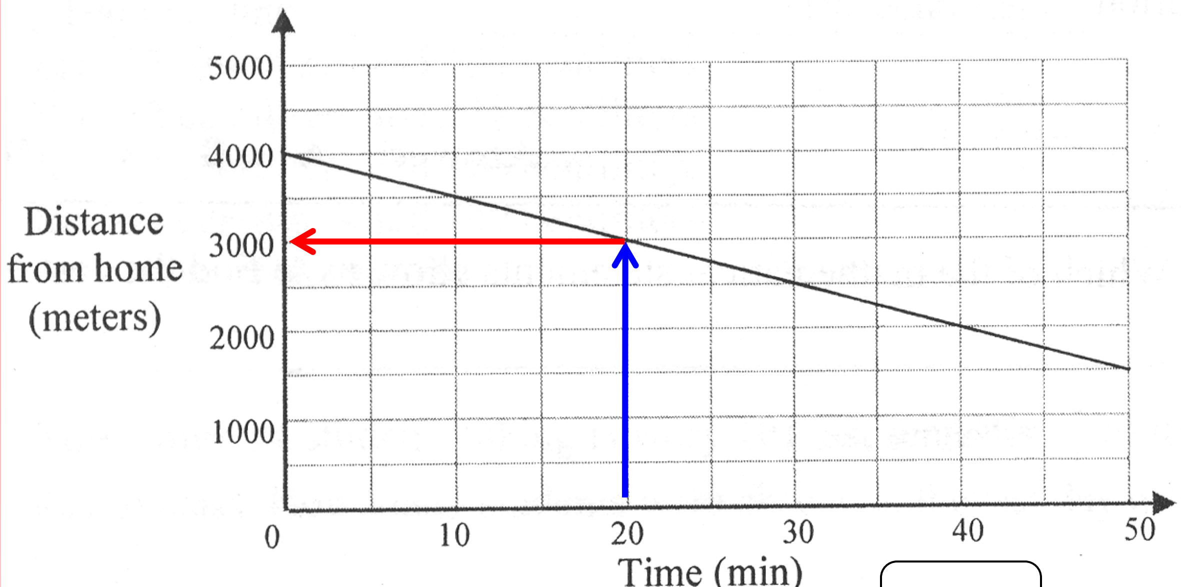
$$(b+4)(b-1)=0$$

$b+4=0$	$b-1=0$
$b=-4$	$b=1$

A 9th Grader - Algebra II

Students' Application of Procedures without Thinking: Evidence #2

. Gina is traveling home from her friend's house. The graph represents a portion of Gina's journey. What is Gina's speed at the 20th minute?



(a) Approximately 3000 meters

28%

(b) Approximately 50 meters/min
(c) Approximately 80 meters/min
(d) Approximately 150 meters/min

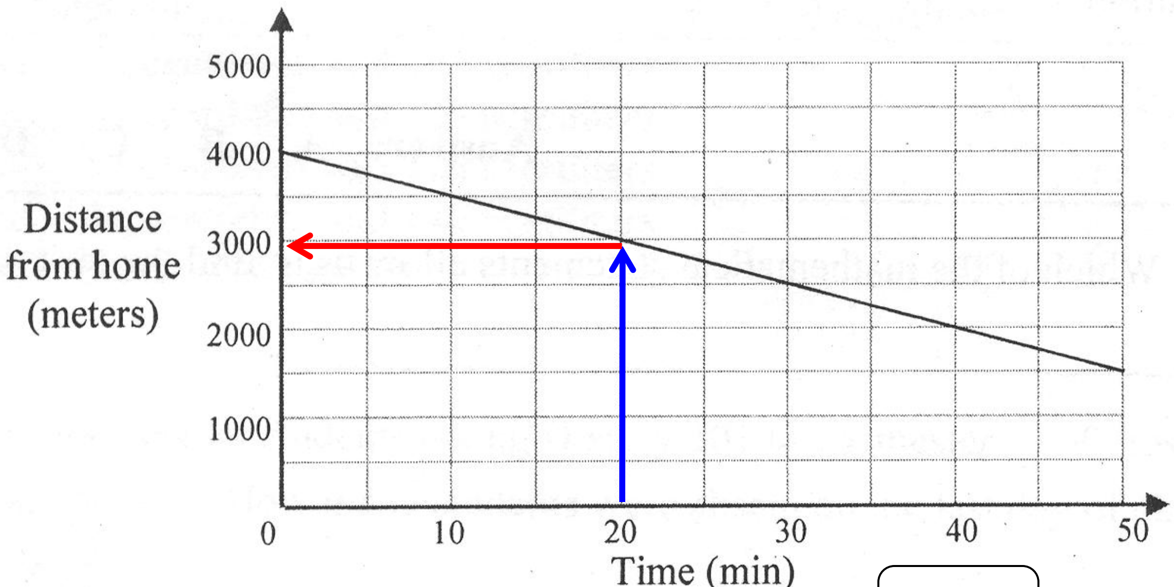
52%

Answer: **A** **B** **C** **D**

307 Pre-service EC-4 Teachers

Students' Application of Procedures without Thinking: Evidence #2

. Gina is traveling home from her friend's house. The graph represents a portion of Gina's journey. What is Gina's speed at the 20^{th} minute?



$\text{speed} = \dfrac{d}{t} = \dfrac{3000}{20} = 150$

$20\overline{)3000}$ 150, 20, 100

(a) Approximately 3000 meters — 28%

(b) Approximately 50 meters/min

(c) Approximately 80 meters/min

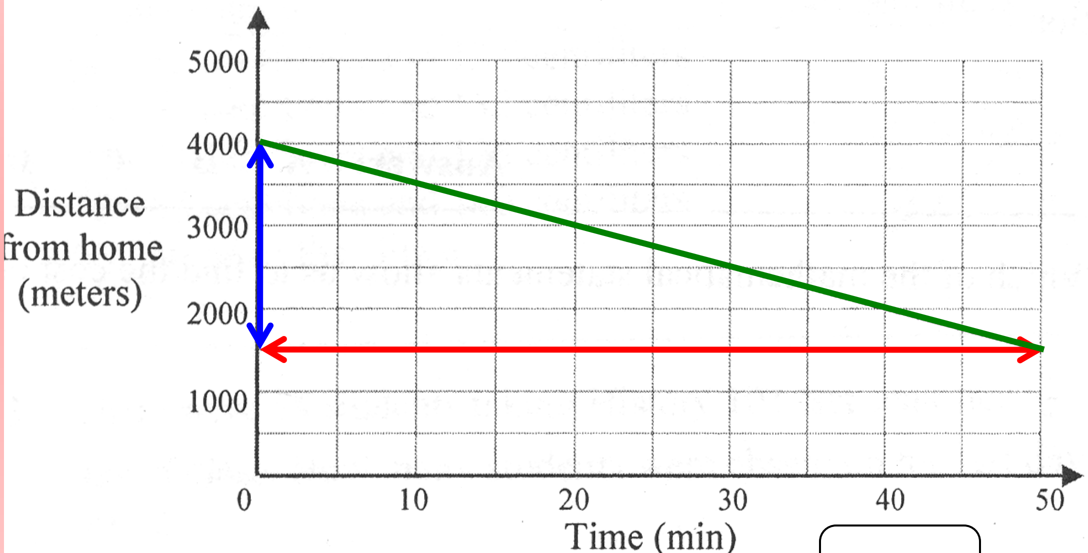
(d) Approximately 150 meters/min — 52%

Answer: **A** **B** **C** **D**

307 Pre-service EC-4 Teachers

Students' Application of Procedures without Thinking: Evidence #2

Gina is traveling home from her friend's house. The graph represents a portion of Gina's journey. What is Gina's speed at the 20th minute?



$\frac{4000 - 1500}{50} = 50$

(a) Approximately 3000 meters — 28%

(b) Approximately 50 meters/min — 18%

(c) Approximately 80 meters/min

(d) Approximately 150 meters/min — 52%

Answer: A B C D

307 Pre-service EC-4 Teachers

The Hammer-and-Nail Phenomenon

"For a person with a hammer, everything looks like a nail"

(A proverb)

The Hammer-and-Nail Phenomenon Exists. So What?

- It reinforces unhealthy beliefs.

  "Doing mathematics means following the rules laid down by the teacher,

  knowing mathematics means remembering and applying the correct rule when the teacher asks a question, and

  mathematical truth is determined when the answer is ratified by the teacher." (Lampert, 1990, p. 31)

- It tells us something important.

  When solving math problems, students are not analyzing.

Two Types of Dispositions

There is a need to advance students …

from Impulsive Disposition

a tendency to proceed with an action that comes to mind without analyzing the problem situation and without considering the relevance of the anticipated action to the problem situation

i.e. tool-oriented

to Analytic Disposition

a tendency to study the problem situation prior to taking actions

i.e. situation-oriented

(Lim, Morera, & Tchoshanov, 2009)

Two Types of Dispositions

Two Possible Explanations to Account for Students' Impulsive Tendency

1. Human Nature
   - Einstellung Effect (Luchins, 1942)

     The phenomenon of solving a given problem in a fixated manner even when a better approach exists.

2. Nurture

Two Types of Dispositions

Two Possible Explanations to Account for Students' Impulsive Tendency

1. Human Nature
   - Einstellung Effect (Luchins, 1942)
   - Dual-process theories
     (Wason & Evans, 1975; Smith, Collins & DeCoster, 2000)
   - Dual-system theories
     (Sloman, 1996; Evans & Over, 1996; Stanovich, 1999)

There are "two distinct cognitive systems, with different structures, functions, and evolutionary histories" (Frankish, 2010, p. 919)

Two Types of Dispositions

Features of the Two Systems

System 1	System 2
Fast	Slow
Automatic	Controlled
Preconscious	Conscious
Low effort	High effort
Heuristic	Analytic
Associative	Rule-based
Implicit	Explicit
Slow acquisition and change	Fast acquisition and change
Parallel	Serial
Does not use working memory	Uses working memory
Independent of general intelligence	Linked to general intelligence
Little variation across cultures	Variable across cultures
Little variation across individuals	Variable across individuals

Source: Frankish (2010)

Two Types of Dispositions

Two Possible Explanations to Account for Students' Impulsive Tendency

1. Human Nature
2. Nurture (School Effect)

> "The tradition has been to regard 'mathematics' as a set of rules for writing symbols on paper, and to regard the 'teaching' of mathematics as merely a matter of 'telling' students what to write and where to write it, together with supervising some considerable amount of drill and practice."
>
> (David, 1989, p. 159)

Two Types of Dispositions

Two Possible Explanations to Account for Students' Impulsive Tendency

1. Human Nature
2. Nurture (School Effect)
    - Compartmentalization of school mathematics
    - Performance-oriented curriculum
    - Clear-and-easy-to-remember instruction
    - Initiate-Response-Evaluate (IRE) interaction

Pedagogical Suggestions

1. Use problem-based learning

Problem-based learning is a teaching method that "consists of carefully designed problems that challenge students to use problem solving techniques, self-directed learning strategies, team participation skills, and disciplinary knowledge"

(Center for Research in Teaching and Learning)

Pedagogical Suggestions

1. Use problem-based learning

How?

One possible approach

- Teacher poses a meaningful problem
- Students work individually
- Students discuss in small group
- Students present solutions
- Teacher orchestrates whole-class discussion, and highlights key concepts and useful habits of mind

Let’s try problem-based learning now!

1. Two identical candles, A and B, lighted at different times were burning at the same constant rate.
When candle A had burned 20 mm, candle B had burned 12 mm.
When candle B had burned 30 mm, how many mm would candle A have burned?

a. Solve this problem?
b. What key mathematical understandings do you want your students learn from working on this problem?
c. What habits of mind do you want your students to develop from working on this problem?

Let’s try problem-based learning now!

1. Two identical candles, A and B, lighted at different times were burning at the same constant rate.
   When candle A had burned 20 mm, candle B had burned 12 mm.
   When candle B had burned 30 mm, how many mm would candle A have burned?

2. Two different candles, P and Q, lighted at the same time were burning at different, but constant, rates.
   When candle P had burned 16 mm, candle Q had burned 10 mm.
   When candle Q had burned 35 mm, how many mm would candle P have burned?

   a. Solve this problem?

   b. Structurally, how is this problem different from the Candle A-B problem?

Think Pair Share

Compare and Contrast

1. Two identical candles, A and B, lighted at different times were burning at the same constant rate.
   When candle A had burned 20 mm, candle B had burned 12 mm.
   When candle B had burned 30 mm, how many mm would candle A have burned?

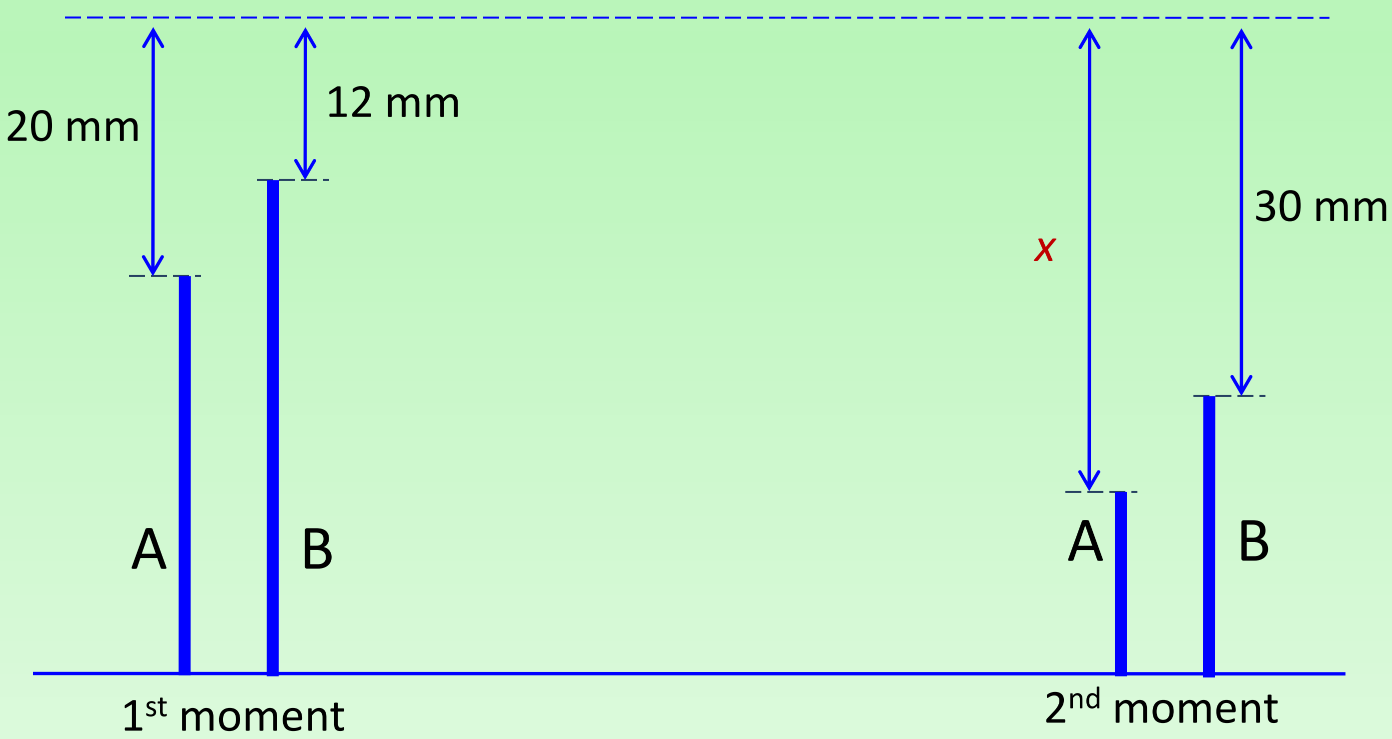
2. Two different candles, P and Q, lighted at the same time were burning at different, but constant, rates.
   When candle P had burned 16 mm, candle Q had burned 10 mm.
   When candle Q had burned 35 mm, how many mm would candle P have burned?

Pedagogical Suggestions

1. Use problem-based learning
2. Include superficially-similar-structurally-different problems
3. Encourage visualizing and drawing diagrams

Visualizing and Drawing Diagrams

1. Two identical candles, A and B, lighted at different times were burning at the same constant rate.
   When candle A had burned 20 mm, candle B had burned 12 mm.
   When candle B had burned 30 mm, how many mm would candle A have burned?



Pedagogical Suggestions

1. Use problem-based learning
2. Include superficially-similar-structurally-different problems
3. Encourage visualizing and drawing diagrams
4. Emphasize quantitative reasoning
   a. Focus on quantities
   b. Focus on relationships among quantities
   c. Focus on meanings of symbols and numbers

a. Focus on Quantities

1. Two identical candles, A and B, lighted at different times were burning at the same constant rate.
   When candle A had burned 20 mm, candle B had burned 12 mm.
   When candle B had burned 30 mm, how many mm would candle A have burned?

Length of candle A burned at the 1st moment

Length of candle B burned at the 1st moment

Length of candle A burned at the 2nd moment

Length of candle B burned at the2nd moment

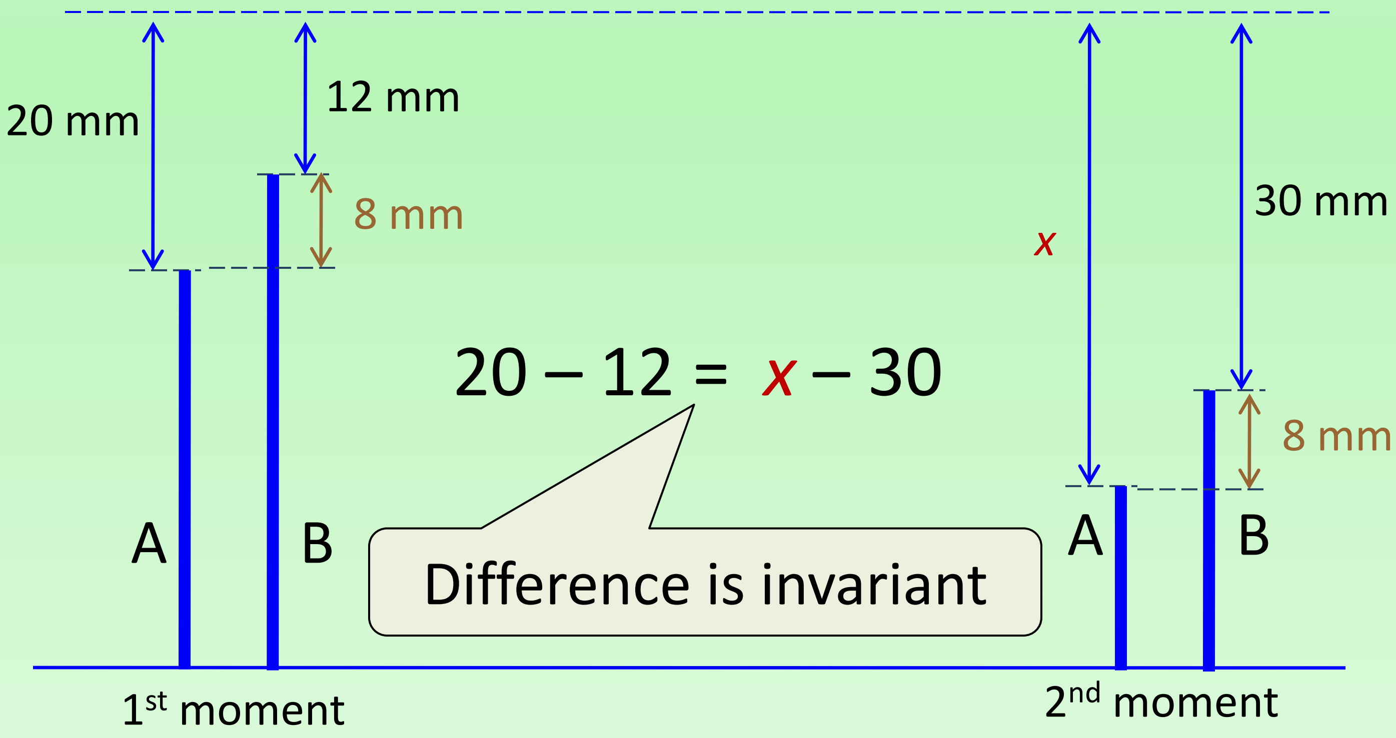
List the quantities.

20 , 12 , and 30

20mm, 12mm, and 30mm

b. Focus on Relationships among Quantities

1. Two identical candles, A and B, lighted at different times were burning at the same constant rate.
When candle A had burned 20 mm, candle B had burned 12 mm.
When candle B had burned 30 mm, how many mm would candle A have burned?



b. Focus on Relationships among Quantities

2. Two different candles, P and Q, lighted at the same time were burning at different, but constant, rates.
When candle P had burned 16 mm, candle Q had burned 10 mm.
When candle Q had burned 35 mm, how many mm would candle P have burned?

Ratio is invariant

$$\frac{16}{10} = \frac{x}{35}$$

c. Focus on Meanings of Symbols and Numbers

2. Two different candles, P and Q, lighted at the same time were burning at different, but constant, rates.
When candle P had burned 16 mm, candle Q had burned 10 mm.
When candle Q had burned 35 mm, how many mm would candle P have burned?

What does 1.6 represent?

$$\frac{16}{10} = \frac{x}{35}$$

Why is x/16 equal to 3.5?

For every 1mm candle Q burn, candle P burned 1.6 mm.

Candle P is burning 1.6 times as fast as candle Q.

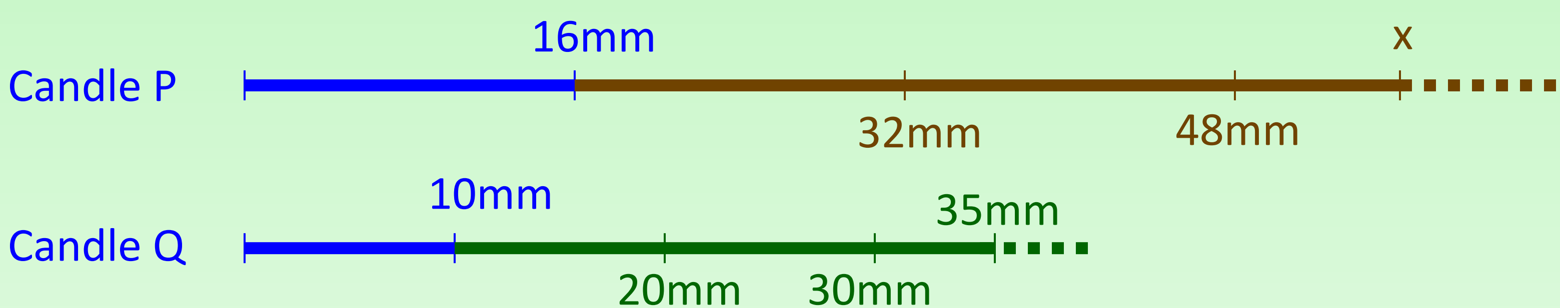
c. Focus on Meanings of Symbols and Numbers

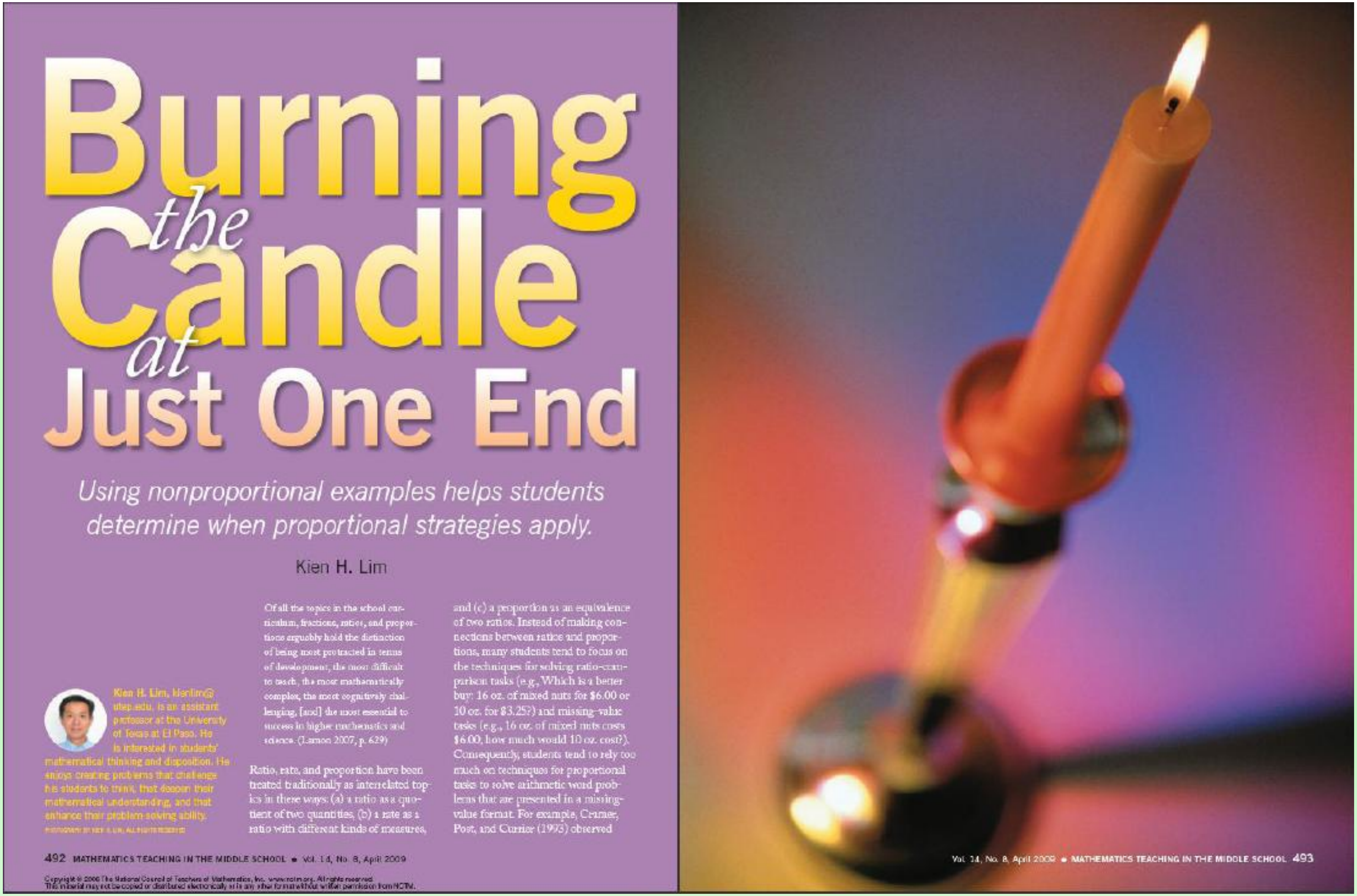
2. Two different candles, P and Q, lighted at the same time were burning at different, but constant, rates.
When candle P had burned 16 mm, candle Q had burned 10 mm.
When candle Q had burned 35 mm, how many mm would candle P have burned?

What does 3.5 represent?

Why is x/16 equal to 3.5?

$$\frac{16}{10} = \frac{x}{35} \qquad \frac{35}{10} = \frac{x}{16}$$





Burning the Candle at Just One End

Using nonproportional examples helps students determine when proportional strategies apply.

Kien H. Lim

> Of all the topics in the school curriculum, fractions, ratios, and proportions arguably hold the distinction of being most protracted in terms of development, the most difficult to teach, the most mathematically complex, the most cognitively challenging, [and] the most essential to success in higher mathematics and science. (Larson 2007, p. 629)

Ratio, rate, and proportion have been treated traditionally as interrelated topics in these ways: (a) a ratio as a quotient of two quantities, (b) a rate as a ratio with different kinds of measures, and (c) a proportion as an equivalence of two ratios. Instead of making connections between ratios and proportions, many students tend to focus on the techniques for solving ratio-comparison tasks (e.g., Which is a better buy: 16 oz. of mixed nuts for $6.00 or 10 oz. for $3.25?) and missing-value tasks (e.g., 16 oz. of mixed nuts costs $6.00; how much would 10 oz. cost?). Consequently, students tend to rely too much on techniques for proportional tasks to solve arithmetic word problems that are presented in a missing-value format. For example, Cramer, Post, and Currier (1993) observed

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Mathematics Teaching in the Middle School

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Pedagogical Suggestions

1. Use problem-based learning
2. Include superficially-similar-structurally-different problems
3. Encourage visualizing and drawing diagrams
4. Emphasize quantitative reasoning
5. Avoid teaching algorithms prematurely

Avoid Teaching Algorithms Prematurely

Two identical candles, A and B, lighted at different times were burning at the same constant rate. When candle A had burned 20 mm, candle B had burned 12 mm. When candle B had burned 30 mm, how many millimeters would candle A have burned?

1° Align info

2° Set up a proportion

3° Cross-multiply

A burned 20, A burned x ?

B burned 12, when B burned 30

$$\frac{20}{12} = \frac{x}{30}$$

$$600 = 12x$$

$$\boxed{50\text{mm} = X}$$

Pedagogical Suggestions

1. Use problem-based learning
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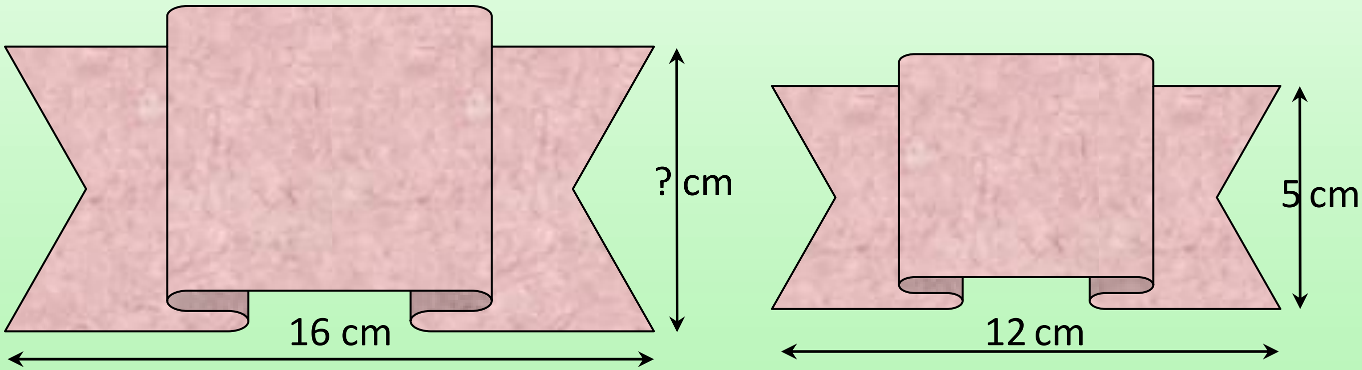
6. Assess conceptual understanding

Assess Conceptual Understanding



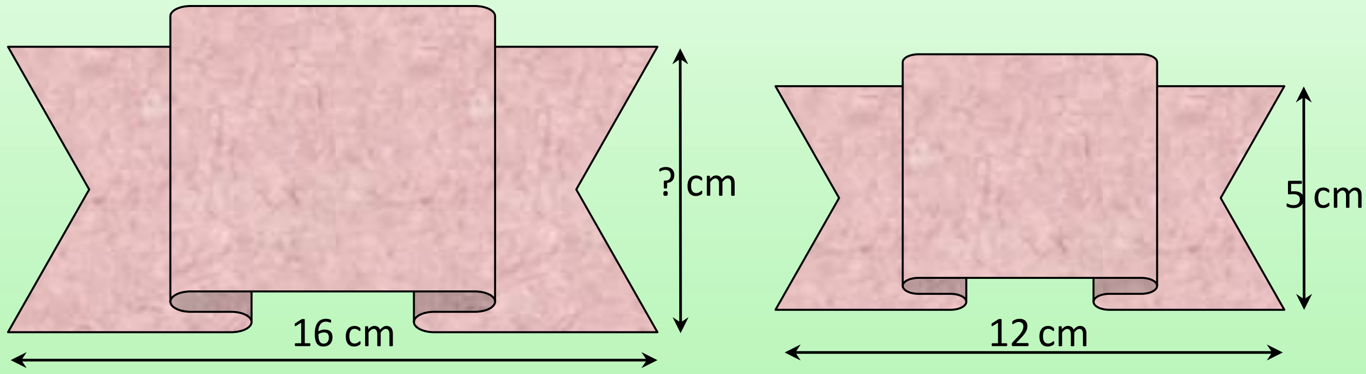
The original picture of a ribbon is shrunk proportionally …
What is the ratio of the breadth of the ribbon in the original picture (left) to the width of the ribbon in the new picture (right)?

(a) 4 : 3 (b) 5 : 6 $^2/_3$ (c) 5 : 9 (d) 9 : 5 (e) None of the above

$$\frac{16 \text{ cm}}{12 \text{ cm}} = \frac{x \text{ cm}}{5 \text{ cm}} \Rightarrow x = 6\,^2/_3$$

But 6 $^2/_3$: 5 is not among the choices.

Assess Conceptual Understanding



The original picture of a ribbon is shrunk proportionally …
What is the ratio of the breadth of the ribbon in the original picture (left) to the width of the ribbon in the new picture (right)?

19%	38%	0%	16%	28%
(a) 4 : 3	(b) 5 : 6 $^{2}/_{3}$	(c) 5 : 9	(d) 9 : 5	(e) None of the above

$$\frac{16\text{ cm}}{12\text{ cm}} = \frac{x\text{ cm}}{5\text{ cm}} \Rightarrow x = 6\,^{2}/_{3}$$

But 6 $^{2}/_{3}$: 5 is not among the choices.

6 out of 32 students chose (a).

Only 2 students chose (a) without any computation.

Pedagogical Suggestions

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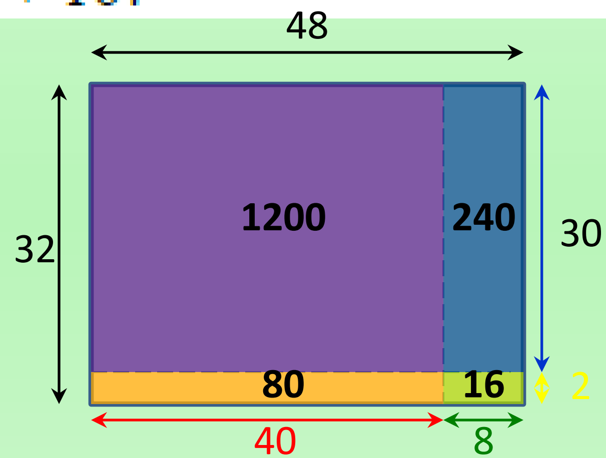
3. Encourage visualizing and drawing diagrams
4. Emphasize quantitative reasoning
5. Avoid teaching algorithms prematurely
6. Assess conceptual understanding
7. Use contra problems in assessments

Use Contra Problems in Assessments (Teach A but Assess A')

An In-class Item

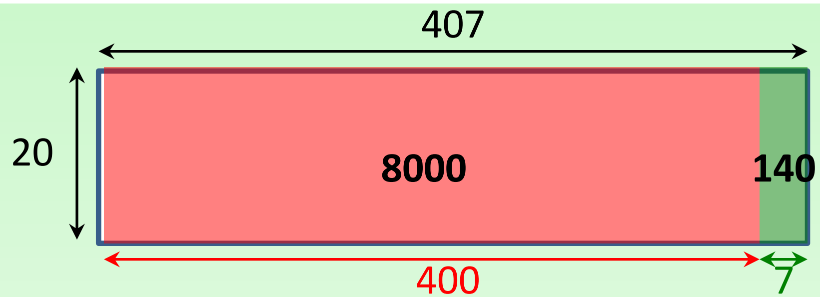
48×32 can be solved by finding the value of $(40 + 8) \times (30 + 2)$.

a. Use the area of a rectangle to show why $(40 + 8) \times (30 + 2)$ is equal to $1200 + 80 + 240 + 16$?



A Mid-Term Exam

Use the area of a rectangle to show that 407×20 is the same as $8000 + 140$.



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Comments from My Students (pre-service 4-8 teachers)

- "I learned to analyze the problem instead of rushing into a procedure, I used to do that."
- "This class helped me … by thinking deeper about that problem instead of just looking at the numbers and wanting to do something with them."
- "In this class, the concepts remain the same, yet the problems themselves are always quite different. I can no longer rely on 'similar problems' in order to figure out my homework or pass [the] exams."
- "This class is very demanding because I have to dedicate more time to learn how to get rid of those 'bad habits' that I have learned in previous classes."

Concluding Remarks

- Students' tendency to apply procedures without thinking is ubiquitous
- Two possible explanations for impulsive disposition
  - Human Nature
  - School Effect
- What should we do?

  Help students progress from an impulsive disposition to analytic disposition
- What can we do?

  Teach in a manner that requires students to think

Thank You