Panel Discussion

"Helping students develop mathematical habits of mind without compromising key concepts from the syllabus"

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Two Habits of Mind
and
How They Can be Developed

- Persistence. This one is widely applicable, even outside of mathematics.

- Writing "Let *x* be a (fixed, but arbitrary) number" into the proof of a universally quantified statement. This one is narrowly applicable.

In his 1985 *Mathematical Problem Solving* book, Schoenfeld provides a framework for “examining what people know, and what people do, as they work on problems with substantial mathematical content” (p. 11). His “four categories of knowledge and behavior” are:

- ***Resources:*** Mathematical knowledge possessed by the individual that can be brought to bear on the problem at hand
  - Intuitions and informal knowledge regarding the domain
  - Facts
  - Algorithmic procedures
  - “Routine” nonalgorithmic procedures
  - Understandings (propositional knowledge) about the agreed-upon rules for working in the domain.

- ***Heuristics:*** Strategies and techniques for making progress on unfamiliar or nonstandard problems; rules of thumb for effective problem solving, including
  - Drawing figures; introducing suitable notation
  - Exploiting related problems
  - Reformulating problems; working backwards
  - Testing and verification procedures

- ***Control:*** Global decisions regarding the selection and implementation of resources and strategies.
    - Planning
    - Monitoring and assessment
    - Decision-making
    - Conscious metacognitive acts

- ***Belief Systems:*** One’s “mathematical world view,” the set of (not necessarily conscious)determinants of an individual’s behavior
  - About self
  - About the environment
  - About the topic
  - About mathematics

(Schoenfeld, 1985, p. 15)

Persistence

One particularly unfortunate belief many students have is: *Any math problem can be solved in 10 minutes or less.*

This can be seen as a cognitive obstacle. If one doesn't persist by spending enough time on a significant problem (not an exercise) at one sitting or come back to the problem repeatedly, one is setting oneself up for failure.

I have taught many sophomore-level transition-to-proof courses. I usually give, at every assessment, both a take-home and an in-class exam. On the final take-home exam, I usually ask students to tell me what they got out of the course – they are not to give me back a list of topics, e.g., sets, functions, induction, etc.

I give the 3 points (out of 100) for this. Students ask how they could get this wrong. I say they only need to give me a thoughtful response.

Of the many responses I have had over the years, the one I particularly liked was, "*I've learned that I can wake up in the middle-of-the-night thinking about a math problem.*"

When a student says this, I know I've accomplished something.

I conjecture that giving students multiple opportunities and motivation

- in the form of take-home tests doable over one week

- along with my telling them that they should start early so they would have enough time

contributed to this student's persistence.

A Narrowly Applicable Habit

Writing *“Let x be a (fixed, but arbitrary) number”* into the proof of a universally quantified statement.

The obstacle here is a lack of feeling, on the student’s part, that writing this matters.

I will illustrate this with the case of Dr. K, a colleague in my department.

Dr. K often teaches beginning real analysis.

When he teaches a topic such as continuity, where the definition reads *for all* $\varepsilon > 0$*, there is a* $\delta > 0$ *such that for all real numbers x …,* he thinks it is important that students learn to take a *fixed, but arbitrary x* (in the proof).

However, as Dr. K (and I) have noticed, students resist doing this – instead, they want to consider *all x* in the proof.

To break them of this (and other) bad habits, he requires that students hand in only 3 proofs per week and grades just 1 proof extremely carefully with copious comments.

Students are given an initial grade on this attempt.

Students are then allowed one additional week to rewrite this proof and hand it in again for more points. The rewritten proof is better and often incorporates Dr. K's suggestions.

One of these suggestions is his insistence that, at the end of such a proof, students write, "*Since x was arbitrary, we have now shown the theorem to be true for all x*."

Thus, students are provided ***multiple opportunities*** and ***motivation*** to consider a fixed, but arbitrary object, as well as write a rationale for this action.

Often, by the middle of the semester, Dr. K no longer insists on this final sentence because, by that time, the students know the reason.

Vignette

John and I learned of this instance through interviewing both our current math ed Ph.D. student, Mary, and Dr. K (as a very small part of a math ed research project we are conducting).

Mary says that when she was taking real analysis with Dr. K, she didn't understand why she was to do this (take a fixed, but arbitrary x), but trusting Dr. K as a representative of the mathematical community with her best interests in mind, and to get a good grade, she did so.

Dr. K thinks Mary did this to get a good grade.

Mary recalls that, part way through the semester, she started making this move automatically, but does not recall feeling differently or understanding the rational for the move.

Now fully two years later, Mary can think of *no other way* to write such proofs.

Moral: It takes a long time to instill good habits of mind. One has to get students to “just do it” **first**. Understanding often follows along somewhat **later**.

How was this habit of mind acquired?

- Repeated practice to get students to "just do it."

- Writing the final required sentence provided students with a rationale for why the theorem had been proved.