MER Special Session on Mathematics and Education Reform at JMM, Washington D.C.

Session: ***Helping Students Develop Mathematical Habits of Mind***
Date: January 6, 2009 (Tuesday)
Time: 1:00 PM – 4:20 PM
Venue: (To be assigned)

Cuoco, Goldenberg, and Mark advocate *habits of mind* as an organizing principle for a mathematics curriculum where students learn to be "pattern sniffers, experimenters, describers, tinkerers, inventors, visualizers, conjecturers, and guessers." Harel regards habits of mind as interiorized *ways of thinking*—conceptual tools that are necessary for constructing mathematical objects. Presenters for this session offer various perspectives and strategies for helping students develop mathematical habits of mind, including examples from different content areas and at different levels.

1. 1:00 PM – 1:20 PM

1046-97-1675 **Kien H Lim*** (kienlim@utep.edu), Department of Mathematical Sciences, University of Texas at El Paso, El Paso, TX 79968-0514. *Undesirable Habits of Mind of Pre-service Teachers: Strategies for Addressing Them.*

In order to help students develop mathematical habits of mind teachers must possess those habits themselves. Many pre-service K-8 teachers, however, enter colleges with undesirable habits of mind such as spontaneously proceeding with an action that comes to mind without analyzing the problem situation, and not attending to meaning of numbers and symbols. Such habits of mind will negatively impact what and how they learn. For example, students tend to focus on procedures for solving problems rather than on their underlying mathematical structures. A study is being conducted to investigate the viability of posing problems for which a recently learned idea would not work to foster the habit of identifying quantities and relationships among quantities. Preliminary results show that habits of mind are resistant to change. (Received September 16, 2008)

2. 1:30 PM – 1:50 PM

1046-97-**** **Paul Goldenberg*** (pgoldenberg@edc.org), Center for Mathematics Education, Education Development Center Inc., 55 Chapel Street, Newton, Massachusetts 02458-1060, USA.
Mathematical habits of mind and the language-learning brain: mathematics as a second language.

Language plays a surprising behind-the-scenes role in elementary arithmetic. The way we name numbers allows certain mathematical ideas to be handled as if they were purely linguistic objects. Language plays a less subtle role in algebra, as algebra is both a set of ideas and a language in which to express them. We use the language aspect of algebra in at least two ways. We describe things that we already know, a semantic use of the language. We also manipulate algebraic expressions according to the syntax of the language to derive things that we did not previously know. For the most part, young children cannot take that second step, using abstract manipulations to derive what they don't know. But they can take the first step, and we too often overlook that descriptive side of algebra that young children can learn easily and excel at. Young children are the best linguists of us all. Given the chance, their linguistic talents can be put to good use in learning mathematics, not only in learning the language side of algebra, but also in understanding number and in performing certain computations. This talk will highlight several mathematical habits of mind that have linguistic roots, and a teaching approach that explicitly takes advantage of those roots.

3. 2:00 PM – 2:20 PM

1046-97-1907 **Hyman Bass*** (hybass@umich.edu), 2413 School of Education, 610 E. University, Ann Arbor, MI 48109-1259. *Algebraic reasoning in 3D, using principles of linearity and symmetry.*

Mathematical habits of mind make use of mathematical connections (for example between algebra and geometry), recognition and use of patterns (such as linearity, symmetry, etc), and multiple (including visual) representations. Yet much of the school curriculum is restricted to shallow and disconnected engagements with these ways of thinking. I propose to show an example of a rich piece of mathematical reasoning and problem solving that integrates all of these features, yet uses only resources available in the middle school curriculum. (Received September 16, 2008)

4. 2:30 PM – 2:50 PM

1046-97-684 **Annie Selden*** (aselden@math.nmsu.edu), Dept. of Mathematical Sciences, New Mexico State University, Las Cruces, NY 88003-8001, and **John Selden** (jselden@math.nmsu.edu), Department of Mathematical Sciences, New Mexico State University, Las Cruces, NM 88003-8001. *Habits of Mind for Proving.*

There are certain aspects of proving that mathematicians do automatically, but that students are often unaware of. We define the formal-rhetorical part of a proof to be those aspects of a proof that can be written by examining the logical structure of the statement of a theorem and by unpacking associated definitions. Examples include writing the first and last lines, unpacking the last line, considering what strategy one might invoke to prove it, and unpacking the hypotheses. Writing the formal-rhetorical part of a proof can expose "the real problem" to be solved. Students need to make doing such things automatic, that is, they must become habits of mind. Beginning students often fail to examine the conclusion; instead, they have a habit of beginning with the hypotheses and forging ahead rather blindly. This may come from high school geometry where this strategy may have served them well. This is a "bad habit" that needs to be replaced by the "good habit" of examining the conclusion. We will discuss this and other habits of mind that can be encouraged such as looking up definitions when one is unsure of what a term means – something students are often reluctant to do. (Received September 10, 2008)

5. 3:00 PM – 3:20 PM

1046-97-1774 **Chris Rasmussen*** (chrisraz@sciences.sdsu.edu), 5500 Campanile Drive, Dept of Mathematics and Statistics, San Diego, CA 92182-7720. *Classroom Norms and Habits of Mind.*

In this report I argue that individual dispositions (or habits of mind) and classroom social and sociomathematical norms evolve together as a dynamic system. In doing so, neither the social nor the psychological perspective is given primacy. Rather, each perspective provides a backdrop against which to consider the other. In particular, I discuss the Cobb and Yackel's (1996) interpretive framework, giving specific attention to student's dispositions and corresponding classroom norms using as an example a university level differential equations class to clarify and illustrate constructs within the framework. The example demonstrates both the normative aspects of the classroom and the corresponding student dispositions, demonstrating how the theoretical constructs of the interpretive framework can be used to explain change in a student's habits of mind. Pragmatically, I argue that one way to give explicit attention to student dispositions in the mathematics classroom is to be deliberate about initiating the negotiation of classroom norms. (Received September 16, 2008)

6. 3:30 PM – 4:20 PM

1046-97-1111 **Kristin A Camenga***(kristin.camenga@houghton.edu), Houghton College, 1 Willard Ave., Houghton, NY 14813, and **Kien H Lim** (kienlim@utep.edu), Dept. of Mathematical Sciences, University of Texas at El Paso, El Paso, TX 79968-0514. *Panel: Helping Students Develop Mathematical Habits of Mind.*

As teachers, we are entrusted to teach the long list of mathematical concepts specified in the syllabus. In addition to the tension between depth and breadth, we need to incorporate into our lessons opportunities for students to develop mathematical habits of mind. According to Cuoco, Goldenberg, and Mark, the goal of a habits of mind curriculum is to help "students learn and adopt some of the ways that mathematicians think about problems." This goal, although challenging, is attainable. In this panel discussion, the presenters for this session will discuss (a) ways for helping students cultivate mathematical habits of mind while learning key mathematical concepts across grade levels from elementary to college, and (b) strategies for addressing the challenges in implementing a habits of mind curriculum. (Received September 15, 2008)