Undesirable Habits of Mind of Pre-service Teachers: Strategies for Addressing Them

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Outline of Presentation

- Habits of Mind (HoM): An overview
- An undesirable HoM: Impulsive disposition
- Strategies for addressing impulsive disposition
- Some encouraging results

Habits of Mind (HoM)

- Organizing principle for mathematics curricula
- General HoM
  - They cut across every discipline
  - E.g. pattern sniffing, experimenting, thinkering, inventing, visualizing, conjecturing, guessing
- Mathematical HoM
  - "the way mathematicians think about problems"
  - E.g. talk big think small, talk small think big, use functions, use multiple points of view, mix deduction and experiment

Cuoco, Goldenberg, Mark (1996)

Ways of Thinking

- HoM are internalized *ways of thinking*.
- Mathematics consists of two complementary subsets.
  - Ways of understanding are the products of doing mathematics, which include axioms, definitions, theorems, proofs, problems and solutions.
  - Ways of thinking are conceptual tools for constructing those products.

- Harel's Duality Principle

"Students develop ways of thinking only through the construction of ways of understanding, and
the ways of understanding they produce are determined by the ways of thinking they possess."

Harel (2007, 2008)

Undesirable WoT

1. Beliefs
   - Mathematics is a collection of rules and procedures.
   - "Doing mathematics means following the rules laid down by the teacher, knowing mathematics means remembering and applying the correct rule when the teacher asks a question, and mathematical truth is determined when the answer is ratified by the teacher."

(Lampert, 1990, p. 31)

Undesirable WoT

1. Beliefs
   - Mathematics is a collection of rules and procedures.

2. Proof-schemes
   - Authoritative proof scheme
   - Empirical proof scheme

(Harel & Sowder, 1998)

3. Problem-solving approaches
   - "Waiting to be told what to do"
   - "Doing whatever first comes to mind … or diving into the first approach that comes to mind"

(Watson & Mason, 2007, p. 207)

Impulsive Disposition

Evidence #1

A group of 5 musicians plays a piece of music in 10 minutes. Another group of 35 musicians will play the same piece of music. How long will it take this group to play it?

$\frac{5}{10} = .5 = \frac{1}{2}$

Cross multiply to figure out since it is proportional

$\frac{5}{10} = \frac{35}{X}$

$5x = 350$

$x = \frac{350}{5}$

$x = 70$ minutes or $\boxed{1\text{ hr } 10\text{ min}}$

Only 47% recognized that the answer is 10 minutes
42% obtained 70 minutes,
and 11% found other numbers.

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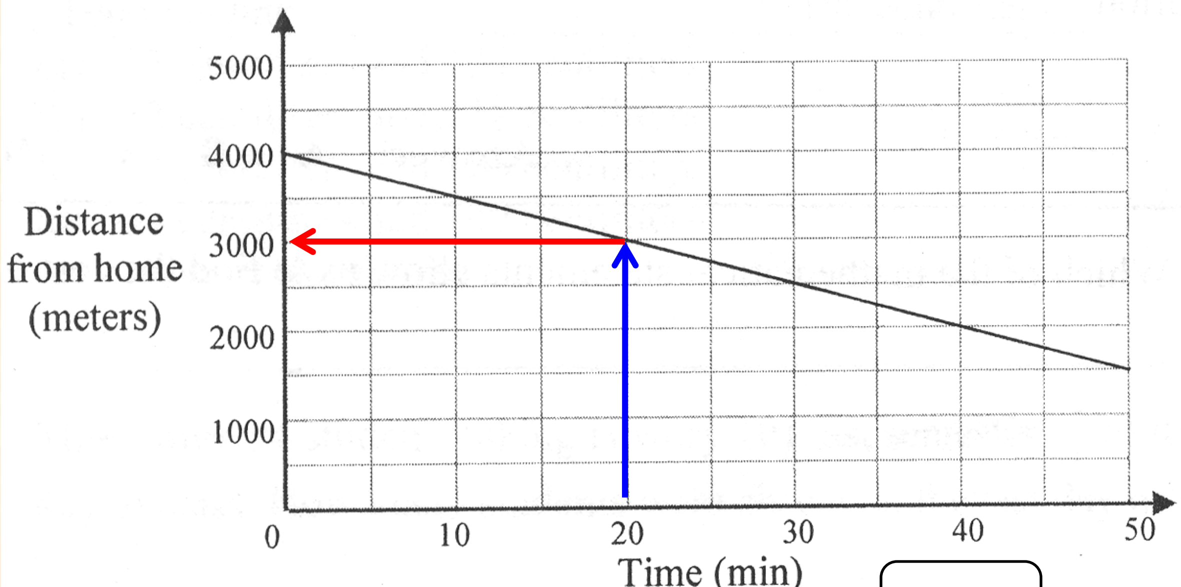
Evidence #2

. Gina is traveling home from her friend's house. The graph represents a portion of Gina's journey. What is Gina's speed at the 20th minute?



(a) Approximately 3000 meters — 28%
(b) Approximately 50 meters/min
(c) Approximately 80 meters/min
(d) Approximately 150 meters/min — 52%

Answer: A B C D

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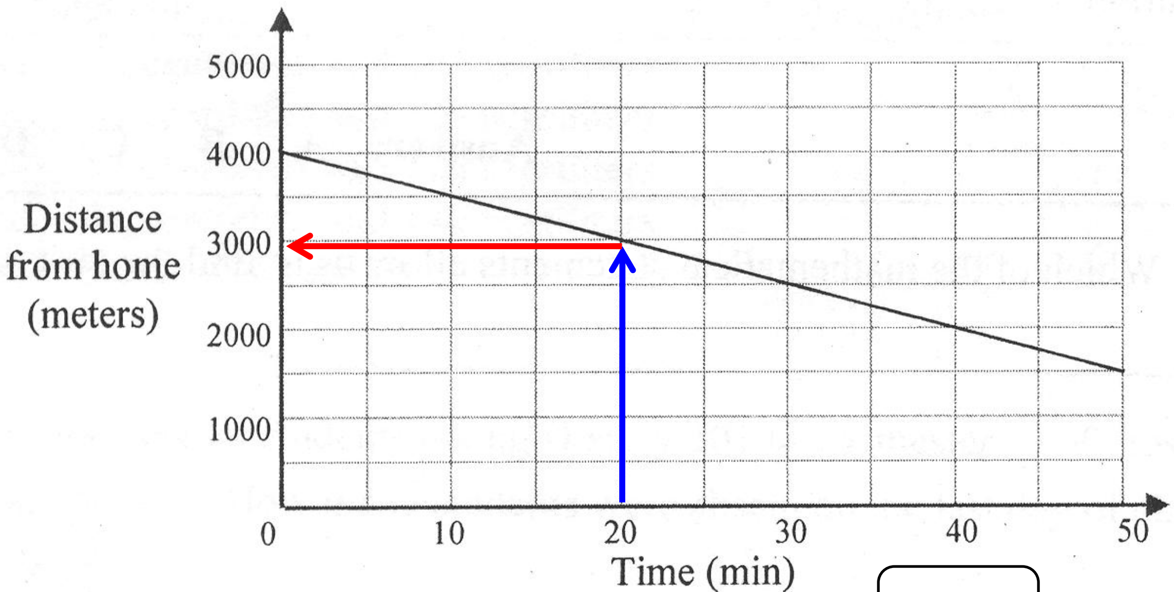
Evidence #2

. Gina is traveling home from her friend's house. The graph represents a portion of Gina's journey. What is Gina's speed at the 20th minute?



$\text{Speed} = \frac{d}{t} = \frac{3000}{20} = 150$

(a) Approximately 3000 meters — 28%
(b) Approximately 50 meters/min
(c) Approximately 80 meters/min
(d) Approximately 150 meters/min — 52%

Answer: A B C D

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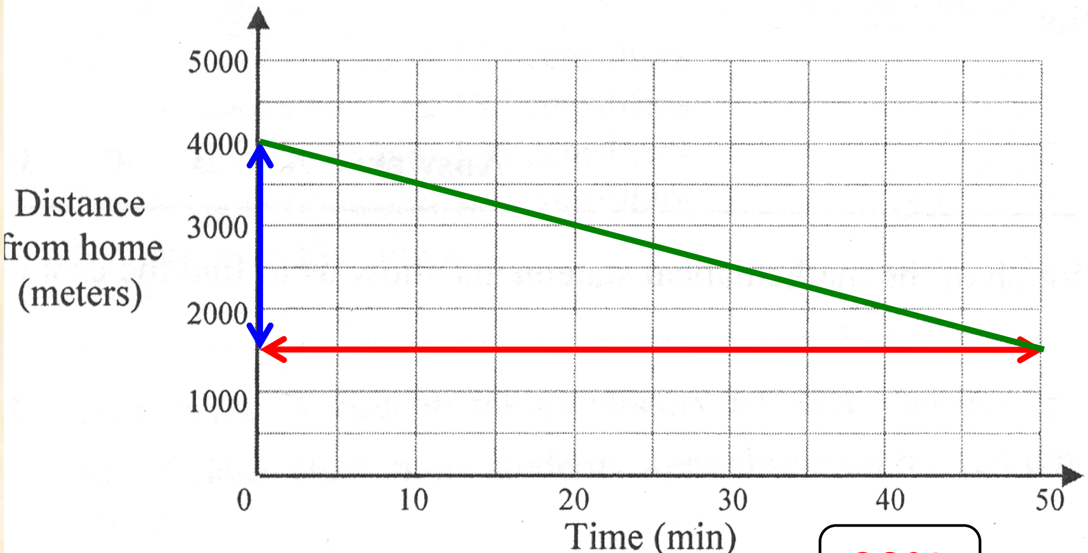
Hammer-and-Nail Phenomenon

Evidence #2

Gina is traveling home from her friend's house. The graph represents a portion of Gina's journey. What is Gina's speed at the 20th minute?



$\frac{4000-1500}{50} = 50$

(a) Approximately 3000 meters — 28%

(b) Approximately 50 meters/min — 18%

(c) Approximately 80 meters/min

(d) Approximately 150 meters/min — 52%

Answer: A B C D

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Two Possible Explanations

- Human Nature

  "Our thinking is canalized with respect to the way we have learned to deal with things … we implicitly anticipate that similar issues have similar causes, and thus similar solutions." (Reigler, 2001, p. 535)

- School Effect (i.e. Nurture)
  - Compartmentalization of school mathematics
  - Emphasis on procedures for solving routine problems

Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept

A new housing subdivision offers rectangular lots of three different sizes:

a. 75 feet by 114 feet
b. 455 feet by 508 feet
c. 185 feet by 245 feet

If you were to view these lots from above, which would appear most square?

(Simon & Blume, 1994)

Practical Suggestions

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- Pose problems that
  - necessitate a particular algorithm/concept

"Students are most likely to learn when they see a need for what we intend to teach them, where by 'need' is meant intellectual need, not social or economic need." (Harel, 1998, p. 501)

Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students

Intriguing Students

Diff = 53

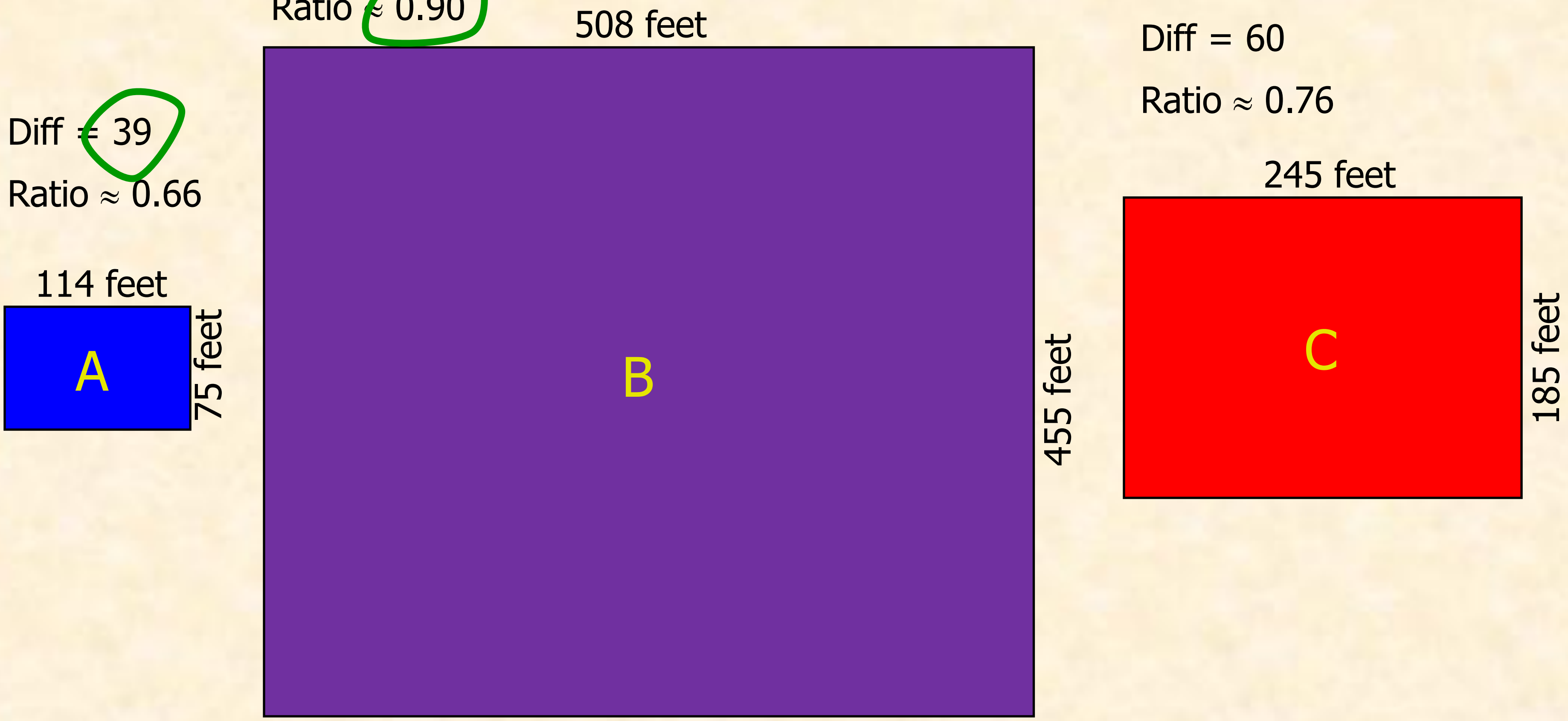
Ratio ≈ 0.90

Diff = 60

Ratio ≈ 0.76

Diff = 39

Ratio ≈ 0.66

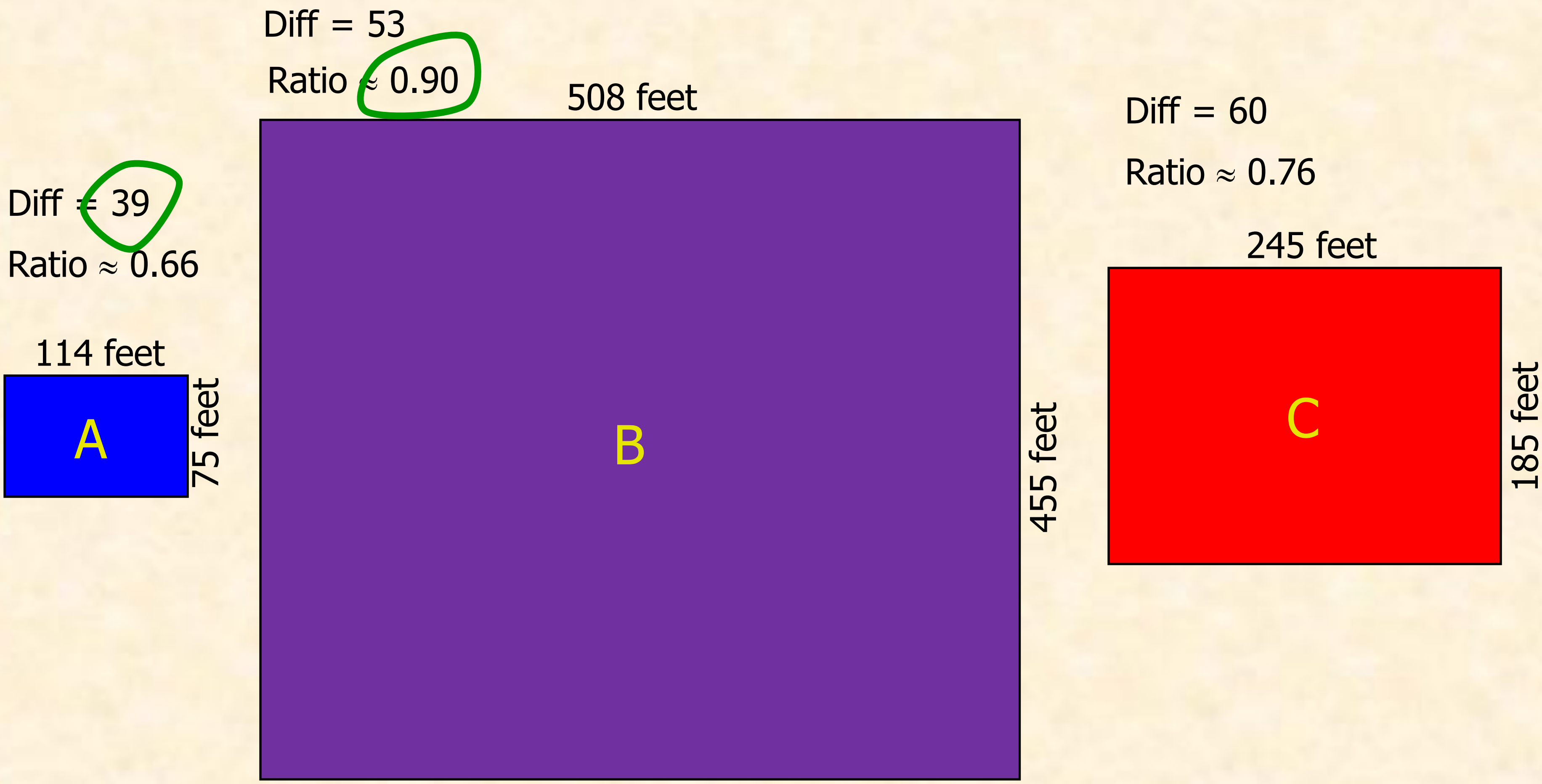


Question: Which method should I use?

Practical Suggestions

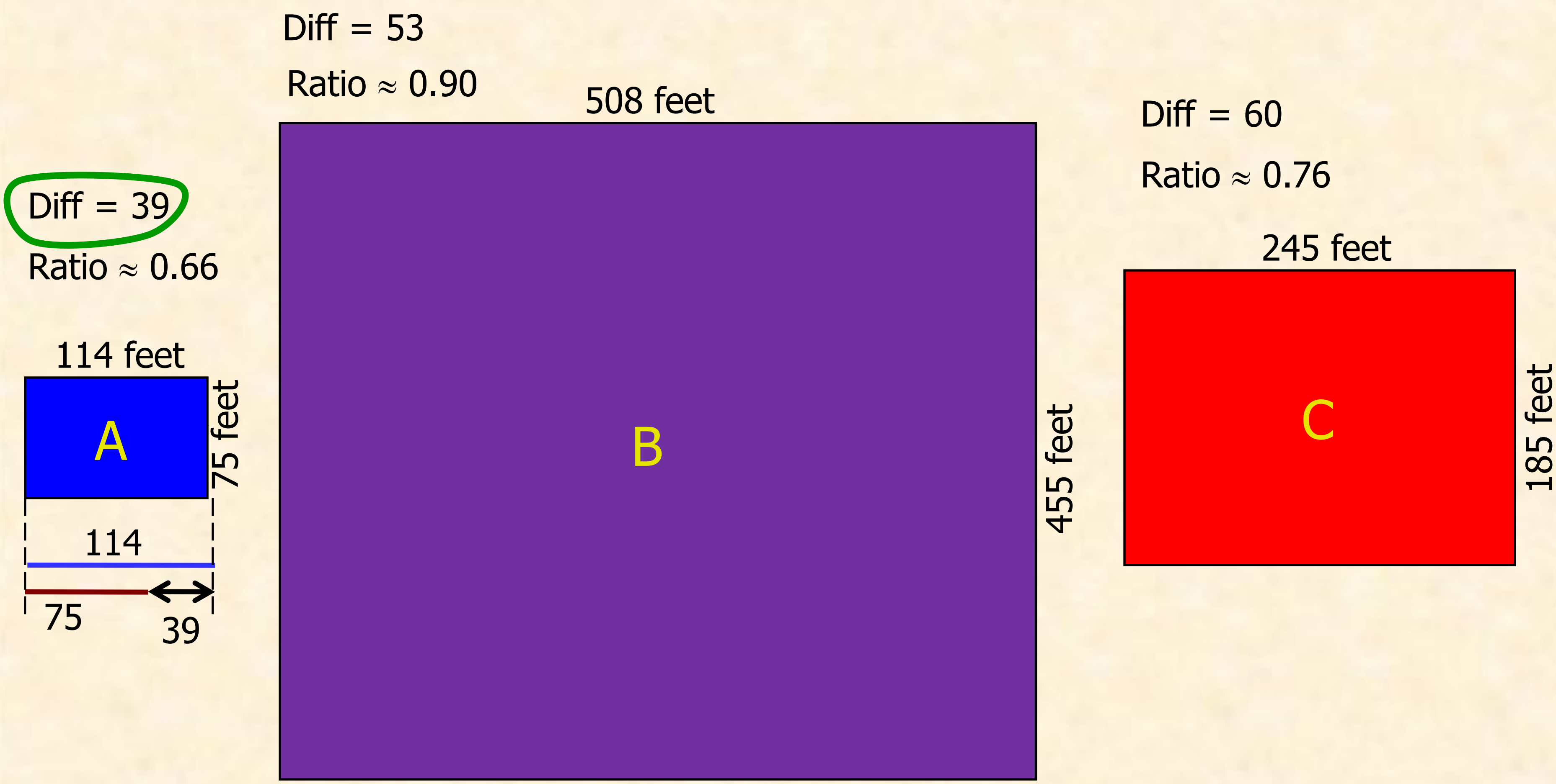
- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols

Attending to Meaning



Question: What does 39 mean? What does 0.90 mean?

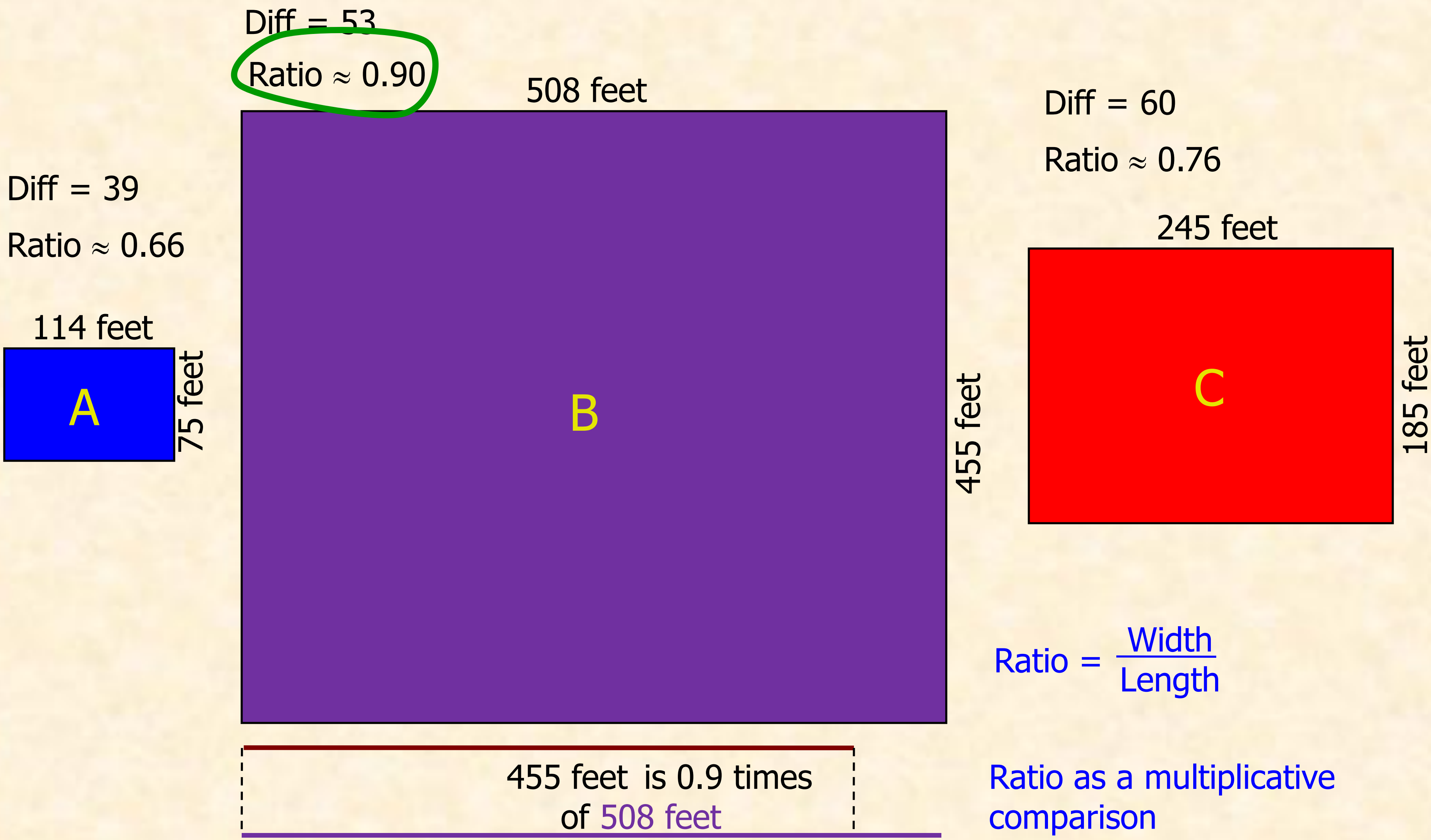
Attending to Meaning



Difference = Length – Width

Question: What does 39 mean?

Attending to Meaning



Ratio = $\frac{\text{Width}}{\text{Length}}$

Ratio as a multiplicative comparison

Question: What does 0.90 mean?

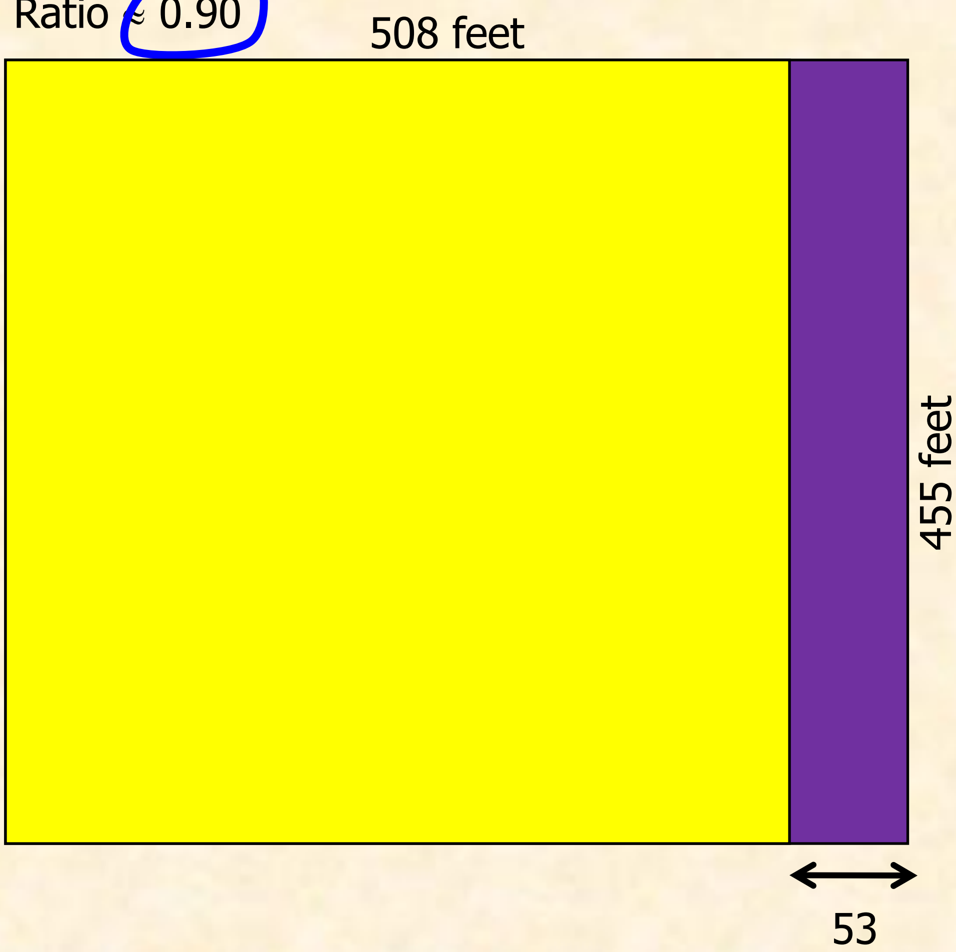
Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify

Explaining & Justifying

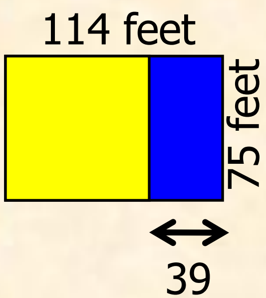
Diff = 53

Ratio ≈ 0.90



Diff = 39

Ratio ≈ 0.66

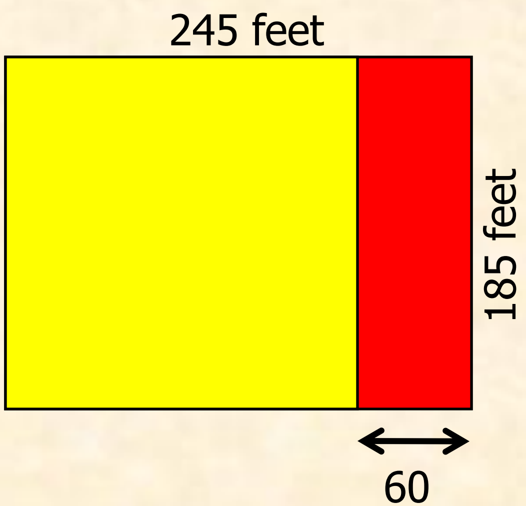


114 – 75 = 39

40 – 1 = 39

Diff = 60

Ratio ≈ 0.76



Difference = Length – Width

$$\text{Ratio} = \frac{\text{Width}}{\text{Length}} = \frac{\text{Width x Width}}{\text{Length x Width}} = \frac{\text{Area of Square}}{\text{Area of Rectangle}}$$

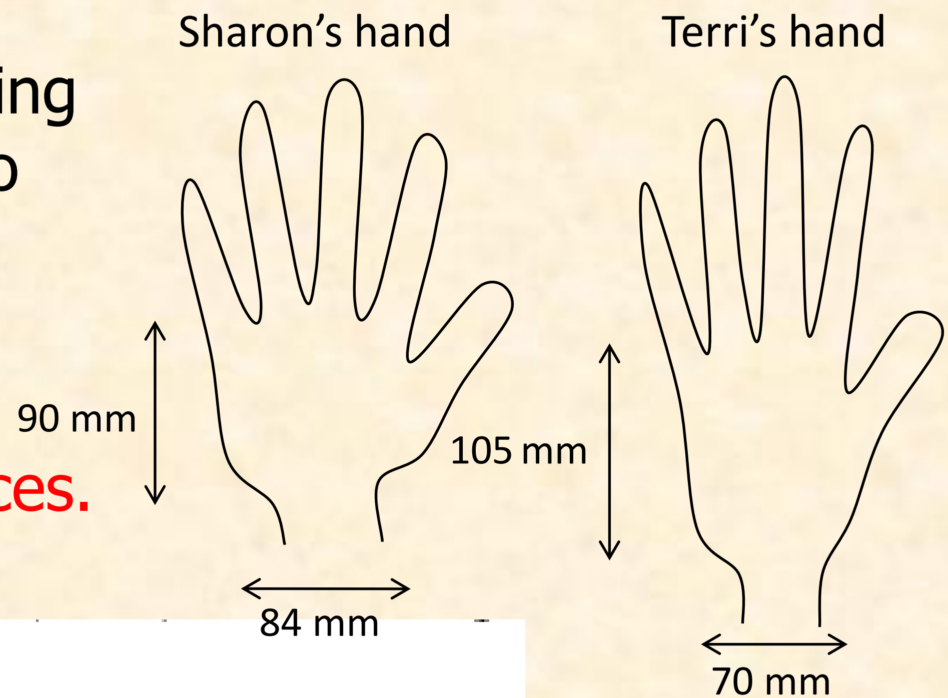
Question: Why is the ratio method better?

Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify
- Include contra-problems to promote skepticism

Promoting Sense-making

Sharon and Terri were comparing the size of their palms. Who do you think has a larger palm?



9 students compared ratios.

3 students compared differences.

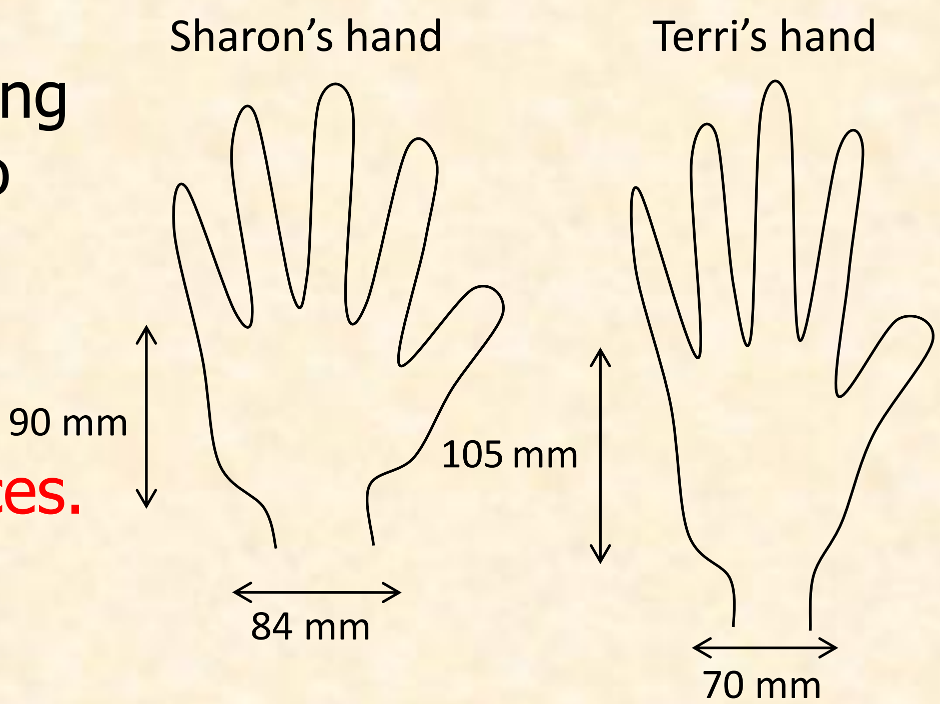
10 students compared areas.

$\frac{90}{84} = 1.071$ $\quad \frac{105}{70} = 1.5$

Terri has a bigger palm because the ratio of her palm height to her width is greater than Sharon's.

Promoting Sense-making

Sharon and Terri were comparing the size of their palms. Who do you think has a larger palm?



9 students compared ratios.

3 students compared differences.

10 students compared areas.

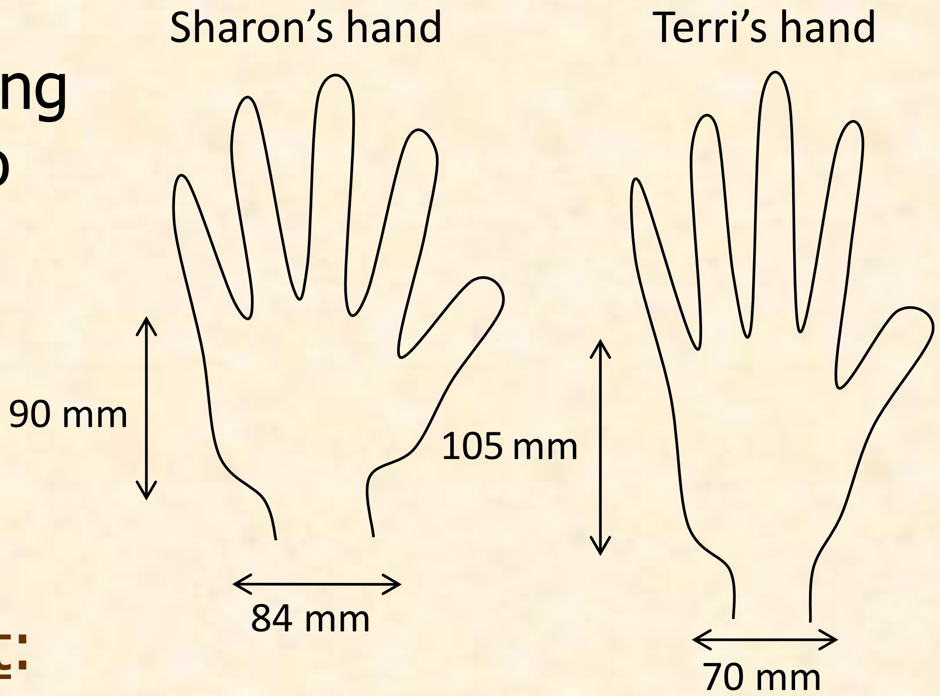
Identifying numbers and selecting operations

versus

Analyzing quantities and relationships

Promoting Sense-making

Sharon and Terri were comparing the size of their palms. Who do you think has a larger palm?



A Student's Written Comment:

"Dr. Lim had the great art of using awesome little tricks that would make us think [that] you [should] use ratios, for example, when in fact it was multiplication! This was a great tactic, because often I would rush right into what I had just been taught, not even looking into the problem."

Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify
- Include contra-problems to promote skepticism
- Include superficially-similar-but-structurally-equivalent problems in tests and exams

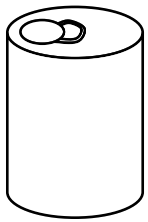
Some Encouraging Results

Fall 07 vs. Fall 08

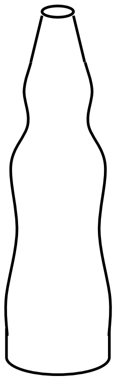
Direct-Proportional Item Fall 07 (47 students) Fall 08 (66 students)

The ratio of the amount of soda in the can to the amount of soda in the bottle is 4:3. There are 12 fluid ounces of soda in the can, how many fluid ounces of soda are in the bottle?

		Fall 07 Pretest → Posttest	Fall 08 Pretest → Posttest
(a)	8 fluid ounces		
(b)	9 fluid ounces	55% ↗ 79%	63% ↗ 77%
(c)	15 fluid ounces		
(d)	16 fluid ounces		
(e)	None of the above		



Can

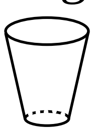


Bottle

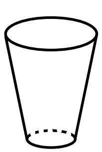
Inverse-Proportional Item

The ratio of the volume of a small glass to the volume of a large glass is 3:5. If it takes 15 small glasses to fill the container, how many large glasses does it take to fill the container?

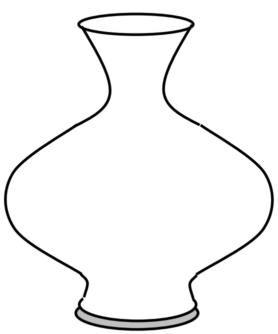
		Fall 07 Pretest → Posttest	Fall 08 Pretest → Posttest
(a)	9 glasses	62% ↘ 43%	34% ↗ 73%
(b)	13 glasses		
(c)	17 glasses		
(d)	25 glasses	28% ↗ 45%	40% ↘ 20%
(e)	None of the above		



Small glass



Large glass



Container

Students' Written Comments

- Do not teach algorithms/formulas prematurely

"My experience in this course was different from that in other classes because in this class … explanation did not come until after we worked on the problem, or after we were assessed. … It has been difficult for me to do math this new way, because I have been taught a different way of doing math for over twelve years. It would take more than just one semester of this kind of math for me to actually make it a habit.

- Include problems that require thinking in quizzes, tests and exams

Students' Written Comments

"I learned to analyze the problem instead of rushing into a procedure, I used to do that."

- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify

"I think that this class helped me … by thinking deeper about that problem instead of just looking at the numbers and wanting to do something with them."

tests and exams

Students' Written Comments

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept

"In this class, the concepts remain the same, yet the problems themselves are always quite different. I can no longer rely on 'similar problems' in order to figure out my homework or pass [the] exams."

- Include contra-problems to promote skepticism
- Include problems that require thinking in quizzes, tests and exams

Students' Written Comments

"This class is very demanding because I have to dedicate more time to learn how to get rid of those "bad habits" that I have learned in previous classes."

"It would take more than just one semester of this kind of math for me to actually make it a habit."

Thank You