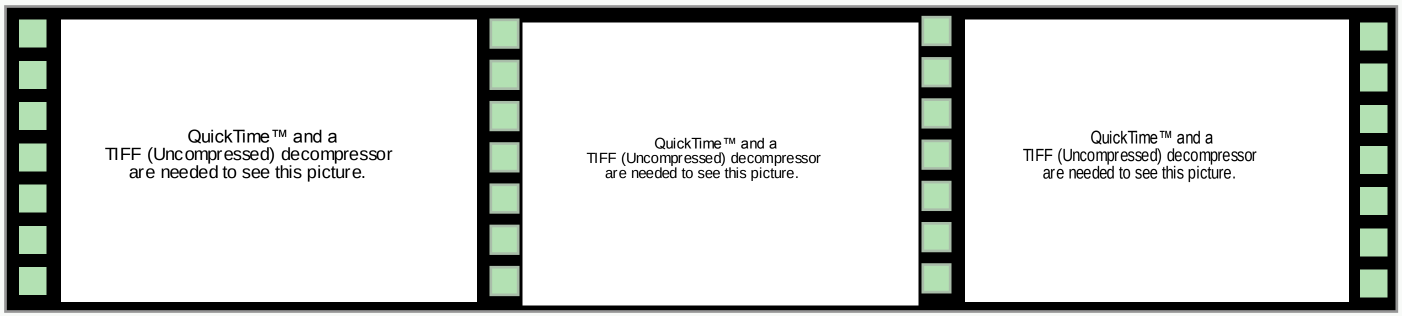
Mathematical Proficiency: An Example from the Inquiry Oriented Differential Equations Project



Chris Rasmussen
San Diego State University
chrisraz@sciences.sdsu.edu

SAN DIEGO STATE UNIVERSITY

Mathematical Proficiency (NRC, 2001)

Mathematical proficiency has five strands:

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

How to positively effect students' mathematical proficiency? Some results and insights from the Inquiry Oriented Differential Equations project.

Inquiry-Oriented Differential Equations

Background theory

- Dynamical systems point of view
- Instructional design theory of Realistic Mathematics Education

Innovative curriculum

- Co-emergent analytic, numerical, and graphical approaches
- Systematic study of student thinking

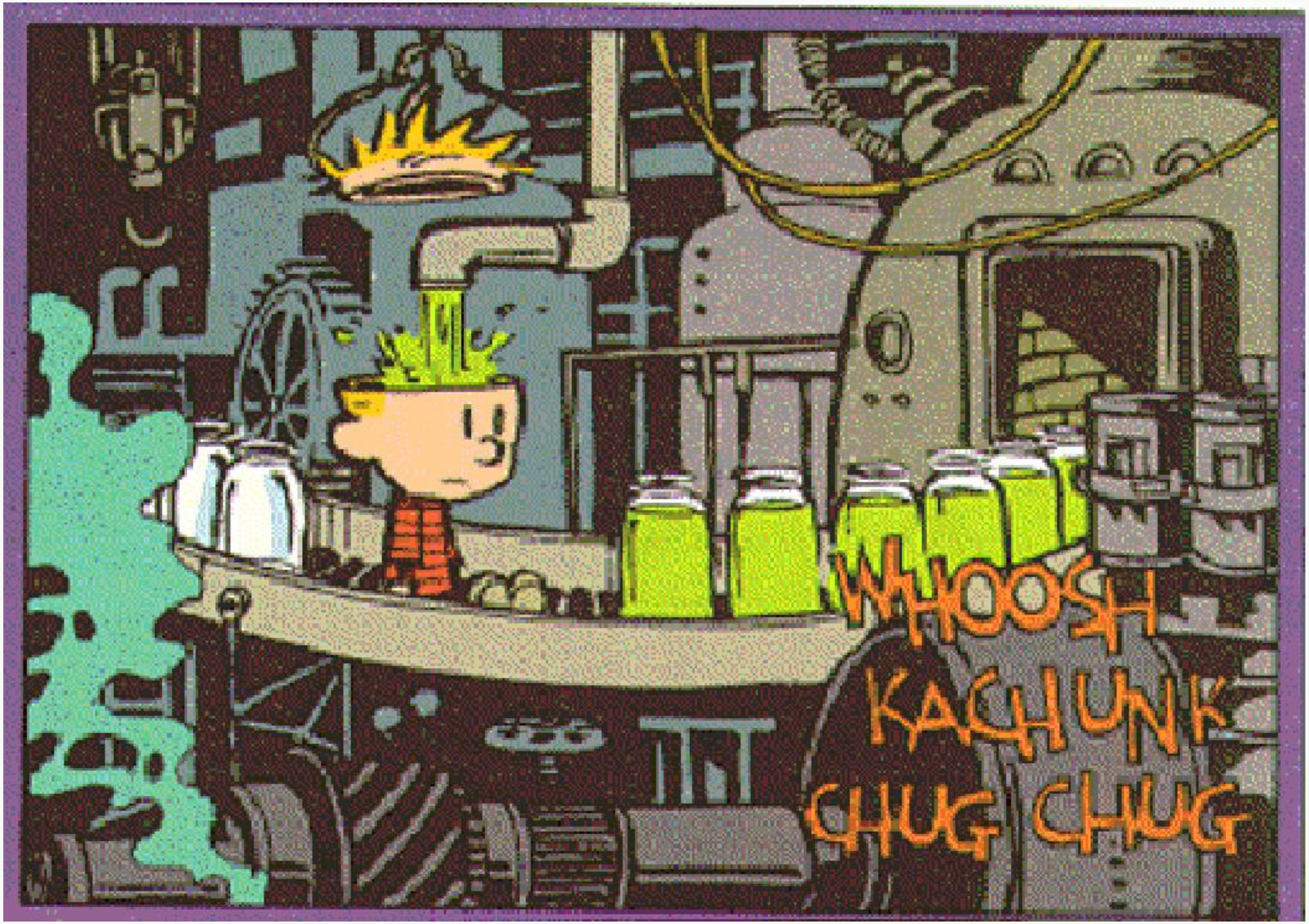
- Java applets

Innovative technological tools

- Social Negotiation of Meaning
- Role of the teacher

Innovative pedagogy

Transmission of knowledge Approach



An Inquiry-Oriented Approach

Student activity

- Students routinely explain and justify their thinking, listen to and attempt to make sense of others' ideas.
- Students engage in genuine argumentation as they (re)invent mathematical ideas.

Teacher activity

- Teachers routinely inquire into how it is that students are thinking about the mathematics.
- Teachers are continually attempting to understand their students' mathematical reasoning.

Systematic investigations of student learning

Quantitative methods (Statistical analysis)	Qualitative methods (Interpretative analysis)

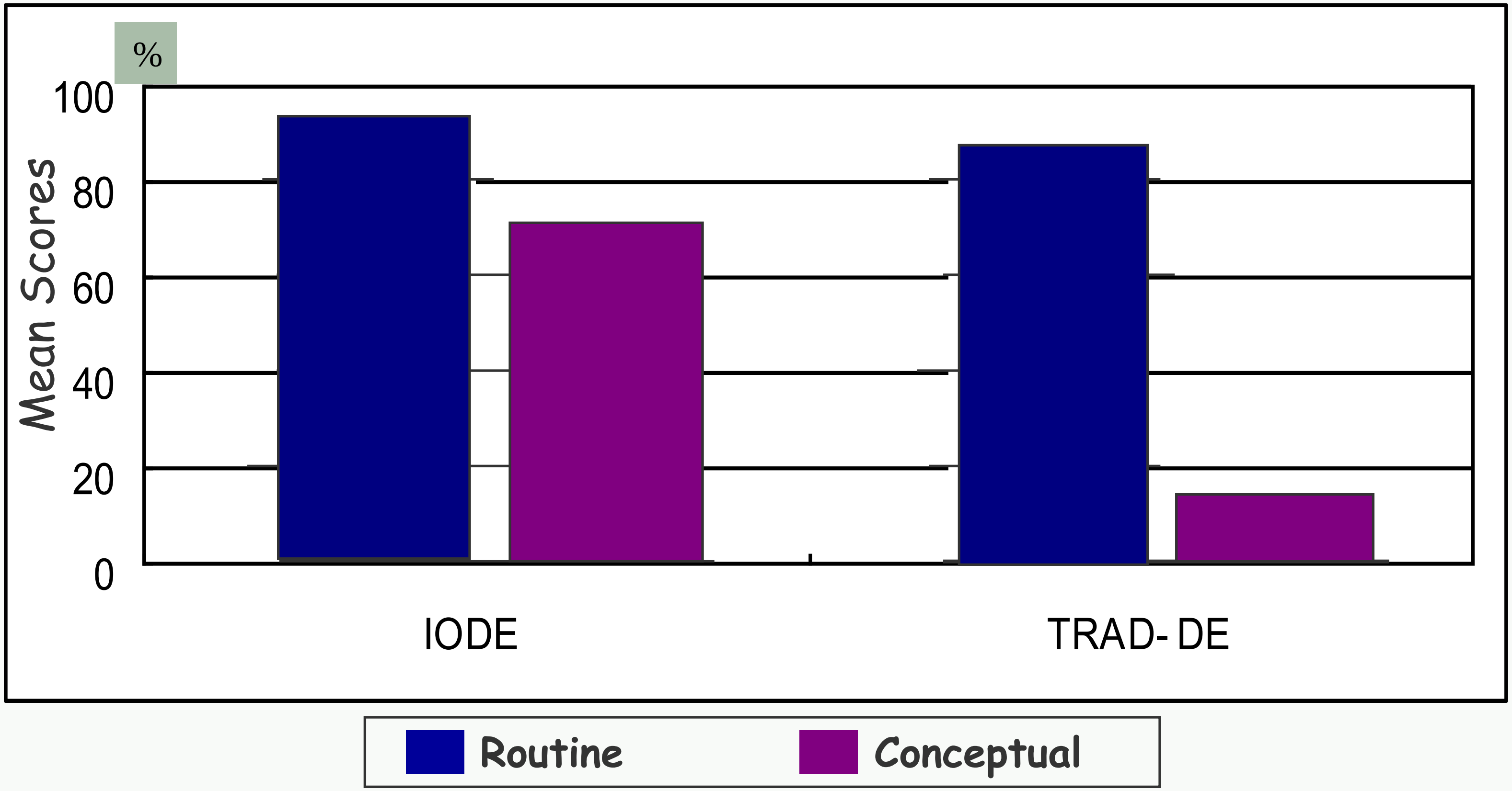
- Individual interview
  - to probe student understanding in depth
- Written questions (pretests and post-tests)
  - to ascertain prevalence of specific difficulties
  - to assess effectiveness of instruction
- Descriptive and explanatory studies during instruction
  - to provide insights to guide instructional design

In the IO-DE project

- Students reinvent many key mathematical ideas (for example, Euler’s method, bifurcation diagrams, analytic technique for solving systems of linear differential equations)

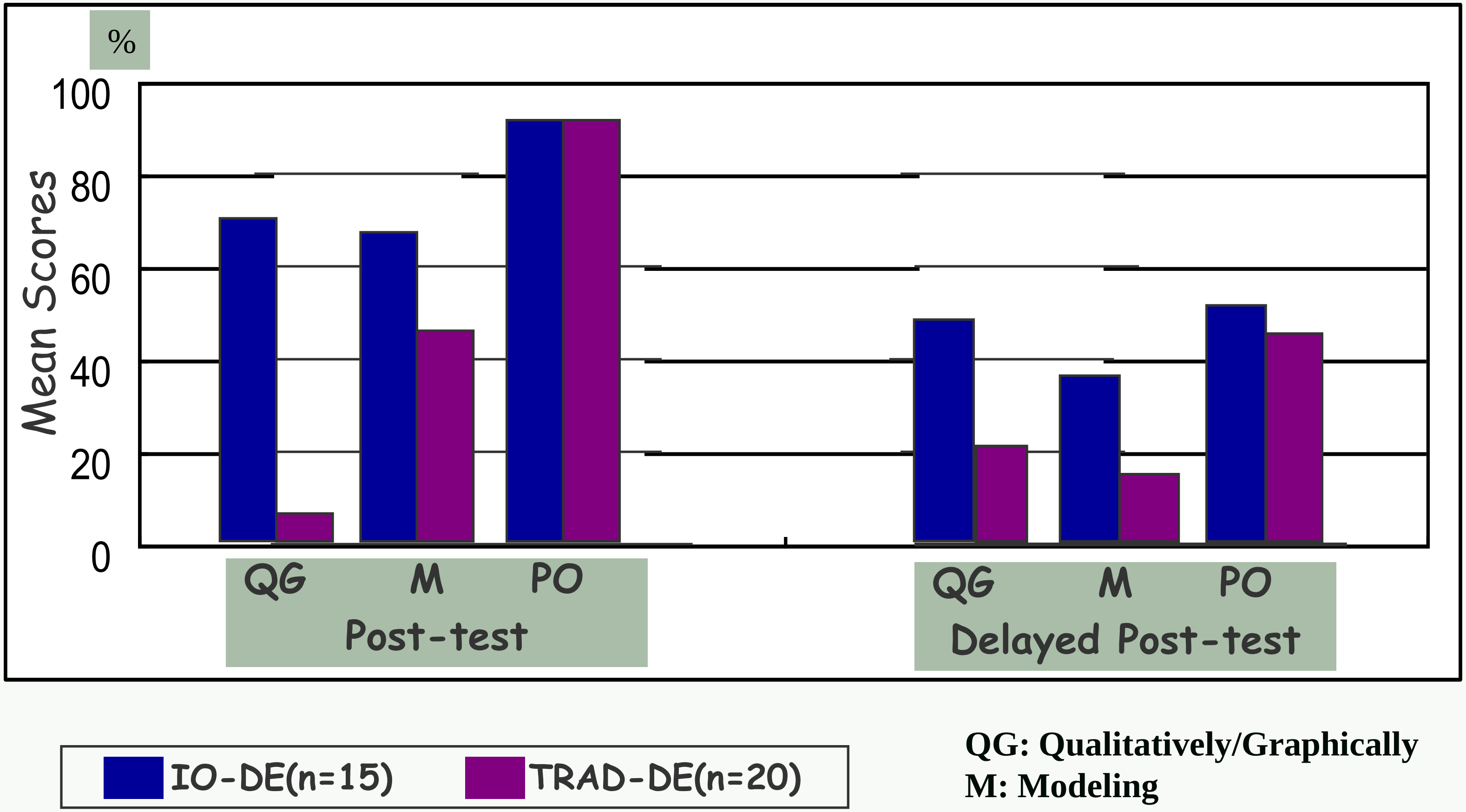
- So what are the benefits for learners? Do they learn ideas more deeply? Do they retain their knowledge longer? Do they develop more productive dispositions about mathematics, about themselves, and their ability to do mathematics?

Assessment of students' conceptual understanding and procedural fluency



- 4 different sites, N = 111

Students' retention of conceptual understanding and procedural fluency



QG: Qualitatively/Graphically
M: Modeling
PO: Procedurally Oriented

Influencing student beliefs and values

- One way to give explicit attention to promoting productive dispositions and habits of mind in the mathematics classroom is for the teacher to be deliberate about initiating and sustaining particular classroom norms
- Some theoretical background on the relationship between classroom norms and beliefs
- Illustrative examples that demonstrate how this theoretical framing can be used to explain change in a student’s beliefs and values

Interpretive Framework (Cobb & Yackel, 1996)

Sociological Perspective (Communities/Collective)	**Psychological Perspective (Individuals)**
Classroom social norms	Beliefs about our own role, othersÕ roles, and nature of math activity
Sociomathematical norms	Mathematical beliefs and values
Collective classroom mathematical activity	Mathematical conceptions and activity

Beliefs can be thought of an individual’s understandings of normative expectancies. Social norms can be thought of as taken-as-shared beliefs that constitute a basis for communication.

Initial Beliefs

J1 - *The only thing that I find sort of confusing is the fact that there may not be an exact answer or answers to a* … *some of these problems that t*… *be used to make educated gu*… *science" where there is always an exact answer or answers to a problem.*

Indicates the student's belief that mathematics should always yield one exact answer was being challenged

J1 - *I'm still getting used to the format* … *everything and not letting the students really have a "voice."*

Belief about the teacher's and the students' roles

J2 - [*M]ost of the* … *answers. I though*… *sufficient to explain my thought process. I now have a better understanding of the expectations.*

This student is articulating that he formerly believed that in mathem… explanation is not required.

P1 - I like the way the class and the book concentrate on practical applications and explanations for differential equations. As you may have noticed from my info card, I have take… *university] and it was* … *all the techniques to* … *was lost and disinterested 15 minutes into the first class session. I can honestly say I think I've learned more about differential equations in the first two weeks here than I did in the whole semester there*.

This student explicitly linked expectations of memorizing techniques to not learning much about differential equations and expectations of explanation to substantive learning.

Negotiating Classroom Social Norms

Excerpts from a twenty-minute episode on day 2

Instructor: Just to sort of recap, last time we were dealing with the spread of a virus. We had [the] elementary school population in Chicago where we had students who were either susceptible to the disease, who were recovere
equation.
fourteent

Shawn likely expected the instructor to evaluate his response as correct or incorrect and then initiate a different question. Instead, however, he calls for additional explanation, in particular, he asks how it "makes sense." In doing so, he indicates that students' responses should explain their individual thinking and further, that mathematical thinking is about sense-making.

Shawn: Fo
the time i

Instructor: Okay, can you explain to us then why it was 1/14 times I? How did that sort of make sense as a way to express the change in the recovered population?

Shawn: That it's constant.

Instructor: Say that again.

Shawn: T
stage.

Instructor: W

With this last question the instructor initiates another shift. His question indicates that he expects others to be actively engaged in the discussion; they are to listen to the exchange he and Shawn are having and are to develop their own interpretations about Shawn's response.

Coordinating Social Norms with Beliefs

Dave: The way I thought about it at first, to make me think that all the points weren't saddles, is that if the next one was a saddle—see how [Bill] has got the one line coming in towards [referring to the phase portrait that Bill had drawn on the blackboard]. Well, if the next one was like that, then you would have to have another point in between those two equilibrium points, like separating, like a source or something. So that

might be a source or maybe a s

Dave's remark elicits the following response from Bill.

Bill: So it's like, you're saying that if there is a saddle, there has to be a source. If there is a sink or a saddle you have to have a, like in this case right here, you would have to have a source in between the saddles in order for it to really make se

These spontaneous remarks made by Dave and Bill indicate that they have taken seriously the obligations of developing personally-meaningful solutions, of listening to and attempting to make sense of the thinking of others, and of offering explanations and justifications of their mathematical thinking. In the process of acting in accordance with these expectations they are demonstrating their beliefs about their roles and about the nature of classroom mathematical activity.

Shortly thereafter, Bil

autonomous first orde

Bill: If you draw t

then you would h

Coordinating Sociomathematical Norms (what counts as an acceptable explanation) and Specifically Mathematical Beliefs

Two aspects of acceptability: (1) when it serves a clarifying function and (2) Grounded in an interpretation of rate of change (versus in terms of procedures).

There is considerable evid students were beginning t students wrote comments

These comments are indications of the students' belief that the purpose of explanations is to communicate; explanations should clarify one's thinking for others.

For example, one student ended his journal with this comment, "This is the best way I know to explain it which I know is very lacking." Another wrote, "I am not completely sure I understand this point so I wouldn't try to explain it to someone unless they had some feedback as to what they think it is."

Instructor: Okay, so Jerry says that if the population gets above 8 they [the fox squirrels] are going to start dying. Tell us why you made that conclusion.

Jerry: Because some number greater than 8 over 8 is going to yield some number greater than one, which 1 minus something greater than 1 is going to give you a negative number and so something times a negative number is going to give you a negative number, so yo

Instructor: And so what d negative, that doesn't tell us anything in itself in relation to the differential equation

Relates to the belief that mathematical explanations have to be grounded in ideas rather than procedures.

Concluding Remarks

- Theoretical comment
  By coordinating individual and collective perspectives, we give primacy neither to the social nor the psychological. Rather, we maintain that each provides a backdrop against which to consider the other.

- Pragmatic comment
  Verschaffel, Greer, and De Corte (1999) have noted that it is generally assumed that students' beliefs about mathematical activity develop "implicitly, gradually, and tacitly through being immersed in the culture of the mathematics classroom" (p. 142). While one generally may agree with this statement, I argue that one way to give explicit attention to student beliefs in the mathematics classroom is to be deliberate about initiating the negotiation of classroom norms. While norms are not rules set out in advance, the teacher is in a unique position to influence the evolution and nature of classroom norms, and hence positively effect student beliefs and values.

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