Mathematical Habit of the Mind for Preservice Elementary Teachers

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Abstract: The concept of Mathematical Habit of the Mind (MHM) urges preservice teachers (PST) to use, in their teaching, such ideas as "Is there a different way to think about this problem?", "What is there that I am not seeing?", possible generalizations, the necessity and use of careful definitions, or dealing with the vagueness of open-ended questions-- basically understanding how mathematicians might think. The work of George Polya has many examples of MHM and MHM is including in the *Mathematical Preparation of Teachers* [CBMS, 2001]. This report will describe the introduction of MHM into the content math course for future teachers through its inclusion in *Mathematical Reasoning for Elementary Teachers* (Long/DeTemple/Millman, 5E) and the reactions of PSTs and nine reviewers of the text (five of whom used the text and four of whom didn't) about the inclusion and emphasis of MHM in preservice teacher education. The attached examples from the book are with the approval of the authors but should not be reproduced without consent from the authors.

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Questions to ask yourself in looking back after you have successfully solved a problem include the following:

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- What was the key factor that allowed me to devise an effective plan for solving this problem?
- Can I think of a simpler strategy for solving this problem?
- Can I think of a more effective or powerful strategy for solving this problem?
- Can I think of *any* alternative strategy for solving this problem?
- Can I think of any other problem or class of problems for which this plan of attack would be effective?

Looking back is an often overlooked but extremely important step in developing problem-solving skills.

Let's now look at some examples.

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8. (Feuding Handshake Problem) There are 100 people in the room, half of whom don't speak to the other half. Assume that if they won't speak to each other, they won't shake hands. How many handshakes are there if everyone shakes hands only once (if they shake at all)?

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9. (Marital Handshake Problem) There are 100 people consisting of 50 married couples in a room. Assuming that no husband or wife shake each others' hand but everyone else shakes hands exactly once, how many handshakes are made?

10. (Your Handshake Problem) Make up a handshake problem similar to the three above.

20. Find a formula for the (positive) difference of the squares of consecutive integers by doing parts (a) and (b).

(a) By way of an experiment, fill in the following table of integers. Do you see a pattern?

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n	1	2	3	4	5	6
n^2	1	4				
$(n+1)^2$	4	9				
difference	3	5				

(b) Using algebra, write a formula for the difference of the squares of consecutive numbers (larger minus smaller), and give a reason why it works.

26. How would you generalize the formula of exercise 20?

(a) If x and y differ by 2, show that the difference of their squares is twice their sum.

$x^2 - y^2 = (x - y)(x + y) = 2(x + y)$

(b) To formulate a generalization of part (a), what condition on the difference between x and y would you suggest? Write a problem which generalizes (a) and prove your result using algebra. ⊡

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DEFINITION *Even and Odd Whole Numbers*

A whole number a is **even** precisely when it is divisible by 2. A whole number that is not even is called an **odd** whole number.

It is useful to examine carefully the implications of this definition, to deeply understand both what it means and what it doesn't mean. As with all definitions, preservice teachers need to know the exact definition and model this precision when working with their future students. For example, as has occurred in the elementary classroom, how would you respond to a child who argues that 6 is even because it is divisible by 2 but is also odd because it is divisible by 3? (see problem 17).

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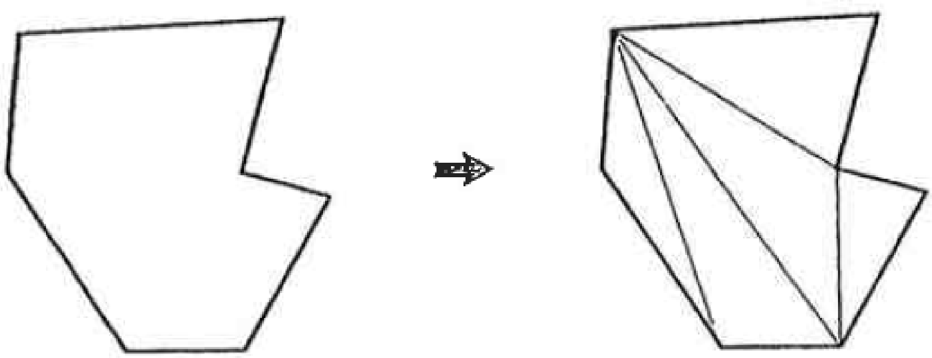
In this section, we begin with ratios and proportions and then apply these concepts to solve problems using proportional reasoning. It is important to understand when and how proportional reasoning can be used as a problem-solving strategy. Of course, it is also important to know when proportional reasoning is inappropriate. For example, it may be tempting to say that a "family-size" 20-inch-diameter pizza will feed twice as many people as the "small" 10-inch pizza. However, using the formula πr^2 for the area of a circle, the respective areas of the pizzas are $\pi(10)^2 = 100\pi$ and $\pi(5)^2 = 25\pi$, so the larger pizza will feed 4 times the number of people as the smaller one. (For more examples of when proportional reasoning is not appropriate, see problems 12, 16, and 17 at the end of the section.)

28. The heptagonal region shown on the left has been broken into five triangular regions by drawing four nonintersecting diagonals across the interior of the polygon, as shown on the right. In this way we say that the polygon is triangulated by diagonals.



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(a) Investigate how many diagonals are required to triangulate any n-gon. $n - 3$

(b) How many triangles are in any triangulation of an n-gon by diagonals? $n - 2$

(c) Explain how a triangulation by diagonals can give a new derivation of the formula $(n - 2) \cdot 180°$ for the sum of the measures of the interior angles of any n-gon. ⊡

33. Consider the triangle SPQ inscribed in a semicircle as shown. Use coordinate methods to prove Thales's theorem; that is, show that $\overline{PQ}$ is perpendicular to $\overline{PS}$. (*Hint:* Recall that $x^2 + y^2 = r^2$ and $(x + r)(x - r) = x^2 - r^2$.)

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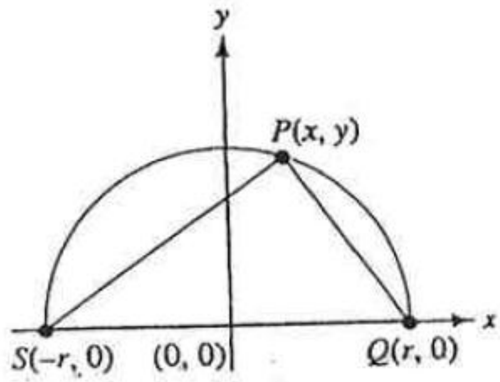

36. Constructing definitions from your intuition and then making sure the definition you've given reflects your intuition is an excellent example of a mathematical habit of the mind. Let's try that approach now. Without looking at another text, give a formal definition, based on your intuition, of the following parts of a circle:

(a) Chord

(b) Diameter

(c) Tangent line

(d) Go to the library at your institution and compare your definitions to those of a high school, middle school, or elementary school text. What are the differences and similarities? Answers will vary.