Undesirable Habits of Mind of Pre-service Teachers

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Outline of Presentation

- Undesirable Ways of Thinking
- Impulsive Disposition
- Pedagogical Suggestions for Addressing Impulsive Disposition
- Some Encouraging Results
- Students’ Written Comments
Undesirable Ways of Thinking

Mathematics

Ways of Understanding

Undesirable WoT

Ways of Thinking

Desirable WoT

Harel (2007, 2008)
1. Beliefs

- Mathematics is a collection of rules and procedures.
- “Doing mathematics means following the rules laid down by the teacher, knowing mathematics means remembering and applying the correct rule when the teacher asks a question, and mathematical truth is determined when the answer is ratified by the teacher.”

  (Lampert, 1990, p. 31)
1. Beliefs
   - Mathematics is a collection of rules and procedures.

2. Proof-schemes
   - Authoritative proof scheme
   - Empirical proof scheme (Harel & Sowder, 1998)

3. Problem-solving approaches
   - “Waiting to be told what to do”
   - “Doing whatever first comes to mind ... or diving into the first approach that comes to mind”
     (Watson & Mason, 2007, p. 207)
An Example

Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20th minute?

Distance from home (meters)

Time (min)

(a) Approximately 3000 meters
(b) Approximately 50 meters/min
(c) Approximately 80 meters/min
(d) Approximately 150 meters/min

Answer: 52%
An Example

Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20\textsuperscript{th} minute?

![Graph showing distance vs. time with points labeled](image)

(a) Approximately 3000 meters
(b) Approximately 50 meters/minute
(c) Approximately 80 meters/minute
(d) Approximately 150 meters/minute

Answer: A

307 Pre-service EC-4 Teachers
Two Possible Explanations

■ Human Nature

“Our thinking is canalized with respect to the way we have learned to deal with things ... we implicitly anticipate that similar issues have similar causes, and thus similar solutions.” (Reigler, 2001, p. 535)

■ School Effect (i.e. Nurture)

- Compartmentalization of school mathematics
- Emphasis on procedures for solving routine problems
Do not teach algorithms/formulas prematurely

Pose problems that
  ○ necessitate a particular algorithm/concept

A new housing subdivision offers rectangular lots of three different sizes:
  a. 75 feet by 114 feet
  b. 455 feet by 508 feet
  c. 185 feet by 245 feet

If you were to view these lots from above, which would appear most square?

(Simon & Blume, 1994)
Pedagogical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept

“Students are most likely to learn when they see a need for what we intend to teach them, where by ‘need’ is meant intellectual need, not social or economic need.”  
  (Harel, 1998, p. 501)
Pedagogical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
Intriguing Students

Question: Which method should I use?
Pedagogical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
**Attending to Meaning**

- **Diff = 39**
  - **Ratio ≈ 0.66**
  - 114 feet
  - 75 feet

- **Diff = 53**
  - **Ratio ≈ 0.90**
  - 508 feet

- **Diff = 60**
  - **Ratio ≈ 0.76**
  - 245 feet
  - 185 feet

**Question:** What does 39 mean? What does 0.90 mean?
Question: What does 0.90 mean?

The ratio 0.90 means that the width of the object is 0.9 times the length. In the context of the diagram, this indicates that the width of object B is 0.9 times the length of object A.

Ratio as a multiplicative comparison:

\[
\text{Ratio} = \frac{\text{Width}}{\text{Length}}
\]
Pedagogical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify
Explaining & Justifying

Question: Why is the ratio method better than the difference method?

Ratio = \frac{\text{Width}}{\text{Length}}

\text{Difference} = \text{Length} - \text{Width}

\text{Ratio} \approx 0.90

\text{Ratio} \approx 0.76

\text{Ratio} \approx 0.66
Do not teach algorithms/formulas prematurely

Pose problems that
- necessitate a particular algorithm/concept
- intrigue students
- require students to attend to meaning of numbers/symbols
- require students to explain and justify

Include contra-problems to promote skepticism
Sharon and Terri were comparing the size of their palms. Who do you think has a larger palm?

- 21% compared ratios
- 16% compared differences
- 13% others (e.g. perimeters, sum)
- 49% compared areas

\[
\frac{90}{84} = 1.071, \quad \frac{105}{70} = 1.5
\]

Terri has a bigger palm because the ratio of her palm's height to her width is greater than Sharon's.

Promoting Sense-making

Fall 08 (61 students)
A Student’s Written Comment:
“Dr. Lim had the great art of using awesome little tricks that would make us think [that] you [should] use ratios, for example, when in fact it was multiplication! This was a great tactic, because often I would rush right into what I had just been taught, not even looking into the problem.”
Pedagogical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify
- Include contra-problems to promote skepticism
- Include superficially-similar-but-structurally-equivalent problems in tests and exams
Some Encouraging Results

Direct-Proportional Item  
The ratio of the amount of soda in the can to the amount of soda in the bottle is $4:3$. There are 12 fluid ounces of soda in the can, how many fluid ounces of soda are in the bottle? 

(a) 8 fluid ounces  
(b) 9 fluid ounces  
(c) 15 fluid ounces  
(d) 16 fluid ounces  
(e) None of the above

Inverse-Proportional Item  
The ratio of the volume of a small glass to the volume of a large glass is $3:5$. If it takes 15 small glasses to fill the container, how many large glasses does it take to fill the container? 

(a) 9 glasses  
(b) 13 glasses  
(c) 17 glasses  
(d) 25 glasses  
(e) None of the above
Some Encouraging Results

Fall 08 (66 students)

Proportional Items (4)
- Week 1: 59%
- Week 3: 69%
- Week 15: 76%

Non-Proportional Items (9)
- Week 1: 44%
- Week 3: 45%
- Week 15: 64%
ASSESSING PROBLEM-SOLVING DISPOSITIONS:
LIKELIHOOD-TO-ACT SURVEY

Kien Lim, Osvaldo Morera, & Mourat Tchoshanov
University of Texas at El Paso

Sep 25, 2009 (8:20am – 9:00am)
Chestnut Room
Students’ Written Comments

■ Do not teach algorithms/formulas prematurely

“My experience in this course was different from that in other classes because in this class ... explanation did not come until after we worked on the problem, or after we were assessed. ... It has been difficult for me to do math this new way, because I have been taught a different way of doing math for over twelve years. It would take more than just one semester of this kind of math for me to actually make it a habit.

■ Include problems that require thinking in quizzes, tests and exams
Students’ Written Comments

“I learned to analyze the problem instead of rushing into a procedure, I used to do that.”

- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify

“I think that this class helped me ... by thinking deeper about that problem instead of just looking at the numbers and wanting to do something with them.”
Students’ Written Comments

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept

“In this class, the concepts remain the same, yet the problems themselves are always quite different. I can no longer rely on ‘similar problems’ in order to figure out my homework or pass [the] exams.”

- Include contra-problems to promote skepticism
- Include problems that require thinking in quizzes, tests and exams
“This class is very demanding because I have to dedicate more time to learn how to get rid of those “bad habits” that I have learned in previous classes.”

“It would take more than just one semester of this kind of math for me to actually make it a habit.”
Thank You