Habits of Mind in the Proving Process

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In the small-group breakout sessions, we hope to find people who will join us in examining things like:

- What do we mean by habits of mind?
- Are there different kinds?
- Are there complex and simple habits?
- Are there large and small habits?
- Can small habits combine to make large ones, etc.?

In case examples might be useful to look at, we will bring a few from:

- A course we are designing to help advanced undergrad and beginning grad math students improve their proving skills.

and

- A “proving skills supplement” for an undergrad real analysis course.

Actions in the Proving Process

We see actions in the proving process as responses to (inner) situations.

After similar situations occur in several proof constructions with the same resulting action, a link is built between the situation and the action.

Example of an action

In a situation calling for *C* to be proved from *A* or *B*, one constructs two independent subproofs arriving at *C*, one supposing *A*, the other supposing *B*.

If one has had repeated experience with such proofs, one does not have to think about doing or justifying this action, one just does it.

We call such persistent (often small grain-size) linked situation-action pairs, *behavioral schemas* and regard them as a special kind of habits of mind.

We see behavioral schemas as a form of (often tacit) procedural knowledge that yields immediate (mental or physical) actions. They call for knowing *how* to act. They are similar to what Mason & Spence (1999) have called “knowing *to* act in the moment.”

On the next few slides, we mention (phenomenal) consciousness.

In saying a person is conscious of something, we mean the person is experiencing it. For example:

- The person might see and understand the equation $5x+2=7$ written on paper.
- The person might hear a sentence spoken.
- The person might “hear” words in inner speech.

Six-point theoretical perspective of the genesis and enactment of behavioral schemas

1. Within a broad context, behavioral schemas are always available – they do not have to be searched for or recalled.
2. Behavioral schemas operate outside of consciousness. One is not aware of doing anything immediately prior to the resulting action.

3. One becomes aware of the resulting action of a behavioral schema as it occurs or immediately afterwards.
4. Behavioral schemas cannot be “chained together” outside of consciousness so that one only becomes aware of the final action. E.g. If the solution to a linear equation would take several steps, one cannot give the answer without being conscious of some of the intermediate steps.

5. An action due to a behavioral schema depends in large part on conscious input.
6. Behavioral schemas are learned through practice. To acquire a schema, a person should carry out the appropriate action (correctly) a number of times. Changing a detrimental schema requires similar, perhaps longer, practice.

(Selden & Selden, 2008)

•*An implication of this perspective*

Because the enactment of a behavioral schema is not under conscious control, conceptual understanding of an error may not be enough to alter a student's behavior.

For example, students who consistently write $(a^2+b^2)^{1/2} = a + b$ may not change their behavior just because they are shown that $(2^2+1^2)^{1/2} \neq 2 + 1$ and agree.

We want to encourage helpful behavioral schemas, or habits of mind, for proving and discourage detrimental ones.

Next we provide an example of a detrimental schema from the literature that we have also seen in our “proofs” class.

Then we will discuss an example of encouraging a student to develop a helpful behavioral schema from a beginning graduate real analysis class .

- ***Focusing too soon on the hypotheses***

  Moore (1994) described undergrad transition-to-proof students who could not prove on the final exam: *If f and g are functions from A to A and $f \circ g$ is 1-1, then g is 1-1.* He said that students started in the wrong place, with the hypothesis, instead of supposing $g(x) = g(y)$, and then using the hypotheses, to conclude $x = y$.

Like (Bob) Moore, we have found that a number of our students habitually focus on the hypothesis immediately, instead of unpacking the conclusion and trying to prove that.

They show a *reluctance to examine the conclusion*, preferring instead to “plunge ahead” by examining the hypotheses immediately.

By patiently guiding students to write the first and last lines of the proof, followed by asking them to unpack the conclusion to “see where they are going,” this detrimental behavioral schema, or bad habit, can be overcome.

- ***Proving universally quantified statements***

One often starts the proof of a statement "For all (numbers) x P(x)" by writing "Let x be a number," meaning x is "fixed, but arbitrary."

Some students are reluctant to write this in their arguments.

Students eventually come to do this as if they were enacting a behavioral schema.

Mary

Mary was a returning grad student taking beginning graduate real analysis with Dr. K, who assigned 3 or 4 weekly proofs, graded them very thoroughly, and allowed them to be resubmitted. He emphasized things like writing "let *x* be a number" into proofs.

Mary recalls feeling this requirement was not particularly important or appropriate. She did so to get full credit.

Near the middle of the course, Mary came to feel that writing things like “let *x* be a number” into proofs “made sense and it was the way to do it.”

She reported to us, two years later, that she cannot think of any other way to write (this aspect of) proofs.

For Mary, this positive behavioral schema took long to develop, but has now become a well-developed habit of mind, with an associated feeling of appropriateness.

(Selden, McKee, & Selden, in press)

Conclusion

We have introduced, and illustrated, a theoretical perspective suggesting that much of the proving process depends on procedural knowledge in the form of small habits of mind, or *behavioral schemas*, some of which are beneficial, while others tend to produce difficulties.

References

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Thank you.

Comments/questions?