

Mathematics Immersion and Habits of Mind:

Preliminary Results from Two Programs

Karen Graham and Todd Abel
The University of New Hampshire

“There are very few absolutes in education, but there’s one thing of which I am absolutely certain: *The best high school teachers are those who have a research-like experience in mathematics.*”

Cuoco (2001)

Mathematics Immersion as a Form of Professional Development

- Based on mathematical exploration – “immerse” educators in mathematical content and practice
- “Genuine” mathematics as practiced by mathematicians
- Typically include a research component or some simulation thereof
 - “Research-like” experience
- We highlight those themes related to habits of mind as described by Cuoco, Goldenberg, and Mark (1996)

“I was not sure what I was looking for or whether I was going down a fruitful path. Normally, this would drive me crazy! And certainly, I had been a little anxious about having such an experience. But now, I feel I have had a glimpse through a window of the world of mathematics.”

*A participant reflecting on their work at the institute

Program #1

A one-week summer institute for faculty members and graduate students responsible for teaching mathematics courses for either prospective teachers or graduate students in mathematics education

- 25 Participants (18 faculty)
- Principal Activities
 - Exploration of problems posed by organizers
 - Individual or small-group investigation of personally-generated mathematical questions
 - Discussion of how such the experience could translate into a mathematics course for prospective teachers or Mathematics Education PhD students
- Analyzed participant journals and follow-up surveys

Themes and Patterns

Freedom to *experiment, conjecture, guess*

- “What this experience has helped me recognize is that teaching mathematics can be doing mathematics, and learning mathematics can be doing mathematics.”
- ”Affectively, working in groups was helpful because I could see that everyone gets discouraged and makes mistakes – the important thing is to learn from the mistakes, as sometimes mistakes help you notice something mathematically!”

Value in *using multiple points of view*

- “Even though I was aware of this skeleton of ideas, as we fleshed them out in discussion I was still making connections and learning why things worked. I could not have this same experience without the collaboration and challenge of additional thinkers.”
- “[T]his ‘problem’ has helped me to do is to feel more confident with considering alternative approaches to examples which have rich algebraic and geometric connections.”

Mathematical work of students

- “It’s so rare in my experience to get to be mathematically creative. It reinforces for me the need to allow our students that opportunity.”
- “Fun Fun Fun! How great it is – AGAIN – a true luxury to be able to play math without the pressure of tests, answers, and rightness, I can see how addictive this is to a learner ...”

“It would be exploring an idea and finding patterns and relationships ... [last year] there would have been less of an emphasis on doing a lot of calculations and more of an emphasis on seeing generalizations, but after this summer I see now that you don’t just, like, look at things, you have to develop them, and so the research part is generating the data and knowing what data to generate and then finding the patterns”

*A participant explaining how they would describe mathematics research to a student

Program #2

A two-summer (6 week sessions) professional development program for high school and middle school teachers housed at a private university

- 50 participants (12 2nd-summer)
- 1st summer: Enroll in a number theory course (large lecture every morning) with problem sets designed to encourage mathematical exploration and habits of mind
- 2nd summer: Re-enroll in number theory course with expectation that work will include more formal proof. Must complete a small group “research project that simulate math research
- Analyzed before and after interviews with 6 participants

Themes and Patterns (Preliminary)

The important motivational role of *pattern-sniffing, experimenting*

- Participants cited the important role of numerical problems in motivating generalizations and conjectures and for inspiring directions for proofs

Mixing deduction and experiment for long-term investigations

- During project work, all participants began with calculations and experimentation, but moved toward deductive proofs of their conjectures
- They frequently returned to numerical experiments

Mathematical work of students

- Classroom practice was of primary importance
- Activities were judged based on their usefulness for classroom work

Observations and Future Directions

Educators immersed in mathematical practices reported increased use and awareness of various habits of mind

The habits of mind that were most prevalent in participant comments were consistent with those stressed by the organizers of the program

Educators are always going to think about their students

Mathematics immersion does seem to impact participants' use and knowledge of habits of mind, but further work needs to be done to understand exactly how