THE NECESSITY OF EQUITY IN TEACHING STATISTICS

Lawrence M. Lesser
The University of Texas at El Paso
Lesser@utep.edu

INTRODUCTION

In a paper on using social justice examples in the statistics classroom, Lesser (2007), it was argued that such an approach goes beyond the "producers and consumers" focus of the national statistics education guidelines of ASA (2007). Nevertheless, it can be packaged as sufficiently mainstream so that even instructors in environments with little precedent may be willing to adopt it. One of the practical strategies offered there to support this approach was to use the term "equity" instead of "social justice" and we adopt that in this paper, with the awareness that the concepts are ultimately inseparable (Lesser, 2009).

Equity has recently gained much attention within mainstream mathematics education in the US, with national organizations issuing position statements, sponsoring featured talks, supporting professional development and coming together for equity summits (Lesser, 2009). Also, there are mathematics education journals with a sustained equity focus (*Teaching for Excellence and Equity in Mathematics*) or that are publishing a focus issue on equity (*Journal for Research in Mathematics Education*; *Journal of Mathematics Teacher Education*).

Lesser (2007) discusses emerging empirical evidence that equity can be a motivating contextual vehicle for engaging students to study mathematics or statistics. For example, in investigating a course for pre-service teachers designed to develop understanding of equity through data-based statistical inquiry, Makar (2004) found significant correlation between their engagement with their topic and the depth of statistical evidence they used, particularly for minority students.

The present paper, however, is articulating a different path from that "equity is a motivational vehicle for learning" empirical approach. We offer a more philosophical argument that can be summarized as follows:

1.) Constructivism says that students' prior conceptions matter.
2.) Students have prior conceptions about fairness.
3.) Concepts about fairness interact or interfere with learning statistics.
4.) Thus, statistics teachers' teaching must acknowledge conceptions about fairness.

NECESSITY

The way we are relying on a more philosophical or logical argument can be viewed as an adaptation of the Harel (1998) "Necessity Principle" to argue that acknowledgment and explicit incorporation of conceptions about fairness is a necessity, not an optional luxury, in the statistics classroom. This would offer a qualitatively different and more inclusive foundation to move towards equity-oriented or critical mathematics education because it would be viewing this type of teaching the domain of not only individuals who already possess a certain orientation such as the public educator as described in Ernest (1991).

Guershon Harel gives the Necessity Principle (NP) as (1998, p. 501):

> "Students are most likely to learn when they see a need for what we intend to teach them, where by 'need' is meant intellectual need, as opposed to social or economic need."

Lim (2009) discusses the reward and difficulty of designing questions or tasks that provoke intellectual need. I postulate that the Necessity Principle has a counterpart for teachers, which we can name the Teaching Necessity Principle (TNP):

> For teachers to teach mathematics or statistics in a particular alternative way, they must have an intellectual need to do so.

PRECONCEPTIONS

Constructivism, a theory and perspective in which learners actively construct their own understanding, has made an impact on statistics education. According to Stern-Dunyak (1993, p. 13): "The 'new' teaching, as described by [David S.] Moore, includes: learning is considered a construction of knowledge; teachers are guides and motivators; students work in groups on open-ended problems; students discuss problems, get feedback; teachers cover less material, but students learn more." Such motivation was present in early experiments such as Shaughnessy (1977, p. 299, emphasis in original):

> A small-group, problem-solving and model-building approach was undertaken in the experimental groups … perhaps the transition for students from preconceptions and misconceptions of probability to mathematizations of probabilistic laws can be facilitated if students are encouraged to experience elementary probability and statistics as a *process* of describing observed experimental phenomena more and more accurately, rather than as a *system* of rules, axioms, and counting techniques that must be learned and applied to problems.

The bottom line is (Shaughnessy 1992, p. 472, emphasis in original): "Our students are not *tabulae rasae*, waiting for the normative theory of probability to descend from our lips. Students already have their own built-in heuristics, biases and beliefs about probability and statistics." This agrees with Mevarech (1983, p. 420): "Evidently,

erroneous schemata are so deeply ingrained in a student's knowledge base that simply being exposed to another statistics course is not sufficient to overcome these errors." Indeed, Shaughnessy (1977) gave students scenarios in which they were not simply told information but had to make predictions, conduct investigations, and then try to reconcile their prediction with the often different outcome.

Students have prior conceptions, however, not only of probability and statistics content, but also of qualities such as fairness. In moving towards building an individual difference measure of conceptions of justice or fairness, PsychWiki (2010) identifies seven types of justice: procedural, distributive (equity), distributive (equality), distributive (need), interactional, informational, and retributional. Beyond the literature on moral reasoning itself, there is research that indicates students have strong ideas of fairness with respect to learning in the classroom, and that these conceptions change with age. Theresa Thorkildsen (1989) interviewed students ages 6-29 and found that older students adopted a more individualistic and meritocratic view towards learning, with respect to five identified levels of conceptions of fairness.

We next consider how the existence of students' prior conceptions of fairness is relevant to how those students learn statistics, and therefore is necessary for teachers to take seriously. The idea that philosophy in general cannot be separated from content is not a new idea. In referencing Lenoir (1979), Dennis (2000) asserts the impossibility of separating Descartes' geometry from his philosophical views in relation to his society.

INTERACTION

Let us consider some ways in which students' concepts of fairness interact with (and may interfere with) learning statistics terminology or concepts, such as: expected value, random sampling, random assignment of treatments, bias, and discrimination.

Annie Svard (2010) conducted a teaching experiment with a class of 27 fourth-grade students in which students were asked to pair up and record the outcomes of 100 trials of a spinner with five equal-sized regions of different colors. She reports that "some students were more concerned with the fairness of the spinner for explaining the variability." (p. 3). This brings up the distinction between whether objects such as spinners and dice are physically fair (e.g., not shaved or weighted) and whether the overall game is mathematically fair (i.e., does one player or "the house" have an edge?). When given the choice between "you cut the cake, I choose" and "you cut, and a coin flip decides who chooses", Chambers (2009) found that his students "overwhelmingly preferred the first solution as 'more fair'" even though the latter option offers an expected value of half the cake no matter how the cake is cut: $(1/2)p + (1/2)(1-p) = ½$.

In discussing Victoria Jacobs' research of upper elementary school children, Shaughnessy (2007, p. 985) reports that, "For many students a survey is not fair unless it has representation from all possible subgroups in the survey population…. this looks like a sophisticated stratified sampling scheme, but in practice those who favor the fair-sample approach would reject any part of randomization." Passionate beliefs for or

against this idea are not limited to schoolchildren, as indicated by this statement in a syndicated column by the economist Thomas Sowell (2010): "One of the biggest fallacies of our time is the notion that if all groups are not proportionally represented in institutions, professions or income levels, that shows something wrong with society." It is conjectured that such students may even find some measures of location more "fair" than others. In particular, would students find the mean of some variable measured for each student more "fair" than the mode or the median because the mean includes *all* data values (and therefore, all students)?

It is not only surveys where student ideas about fairness can interfere. Students' concepts of fairness may affect how readily they embrace the role of randomness in assigning treatments to participants in experiments. Vogt (2007) presents a counterargument some students may believe that treatment resources should be assigned to the neediest students or patients, not the luckiest.

A student's focus on fairness can interact or interfere with meaning of statistical content (Lesser 2010), even on a conceptually abstract level, especially if it is not explicitly acknowledged and distinguished. For example, a student's prior familiarity with the word *bias* as it pertains to a (prejudiced) person may interfere with that student applying it to a situation (e.g., nonresponse) or to an estimator (where the bias is often something that can be computed). This lack of clarity may make it hard for a student to distinguish between disparate treatment and disparate impact, the latter of which has various associated quantitative approaches. For a more advanced example, a student's prior familiarity with the word *discrimination* is usually only with its negative meaning of prejudice against a person or group. This may interfere with embracing the neutral usage of statistics in which discriminant analysis separates items into two or more categories or where the discrimination index of a test item measures the degree to which the test's high-scorers perform better on that item than the low-scorers.

DISCUSSION

It would be interesting to explore possible relationships between a student's concept of fairness (assessed from ideas such as some in PsychWiki, 2010) from that same student's type or level of probability and statistics reasoning. Towards the latter, Shaughnessy (1992, p. 485) states that college students (especially those in introductory courses) usually have one of the two lowest types of conceptions of stochastics: non-statistical ("responses based on beliefs, deterministic models, causality, or single outcome expectations; no attention to or awareness of chance or random events.") or naïve-statistical ("use of judgmental heuristics, such as representativeness, availability, anchoring, balancing; mostly experientially based and nonnormative responses; some understanding of chance and random events"). We conjecture that there is a relationship between students who are quick to declare bias on the basis of a single incident and students who have a schema Konold calls the "outcome approach", in which students perceive each trial of an experiment as "a separate, individual phenomenon" rather than "embedded in a sample of many such trials" (Shaughnessy, 1992).

On another front, it would be interesting to explore the instances for which taking into account cultural context is necessary when teaching statistics. A great example happens to be one of the most widespread analogies in the mathematical sciences. The central focus of statistics is hypothesis testing and the judicial analogy is arguably the most commonly used one in statistics textbooks and is also the most developed analogy in the collection of Martin (2003). And yet, this analogy is founded upon the idea that the default assumption (i.e., null hypothesis) in the courtroom is "innocent (unless/until proven guilty)", which is the *reverse* of the default assumption in countries with a Napoleonic Code of law. As part of a quantitative follow-up to Lesser & Winsor (2009), this question was put to the test in a survey administered to college students the first day of the fall 2009 semester in the five sections of a statistical literacy course at a Hispanic-serving institution on the US-México border:

> In statistics, the "null hypothesis" is what we assume is true until there is significant evidence found against it. What would you say is the null hypothesis for a trial in a court of law?
> a) the defendant is innocent
> b) the defendant is guilty
> c) it could be either of the above, depending on what culture you are from
> d) I do not understand the question

On this item, the 52 English language learner students in the sample had a significantly different ($p = 0.006$) response pattern from the 83 native English speakers in the sample:

	Choice a	Choice b	Choice c	Choice d
English language learners	20	9	13	10
Native speakers	46	11	24	2

While this result is very suggestive of the effect of culture, it should be kept in mind that the proxy of language for culture is not a perfect one. In any case, the nontrivial percentage of students in the entire sample that had an intuition counter to the "textbook perspective" should make teachers take notice. It may not be clear whether the better course of action is (1) to replace the courtroom scenario with something else (e.g., medical testing scenario) or (2) actively make it a teachable moment by discussing that null and alternative hypotheses are *chosen* and that there is asymmetry in that choice and also between Type I and Type II errors. What is clear, however, is that it is dangerous to proceed with the lesson simply assuming all students understand the example as presented in the textbook.

Abdelbasit (2010, p. 2) relates challenges of his Oman students trying to relate to the culture in the American textbooks his university generally uses: "Students do not have a feel for baseball, golf, opinion polls, stock market shares, advertising research, dog sled racing, etc. and examples involving topics like belief in afterlife, abortion, pre-martial sex, etc. are simply off limits….The problem is worse in introduction to probability where games of chance are extensively used as illustrative examples." Abdelbasit found that such foreign context contributes to students losing interest and focusing mostly on mechanics at the expense of conceptual understanding or interpreting results in context.

REFERENCES

Abdelbasit, K. M. (2010). Teaching Statistics in a Language Other Than the Students'. In C. Reading (Ed.), *Data and Context in Statistics Education: Towards an Evidence-Based Society. Proceedings of the Eighth International Conference on Teaching Statistics*. Voorburg, The Netherlands: International Statistical Institute. http://www.stat.auckland.ac.nz/~iase/publications.php

American Statistical Association (2005). *Guidelines for Assessment and Instruction in Statistics Education: College Report*. Accessed September 7, 2010 at http://www.amstat.org/education/gaise/.

Chambers, T. (2009). A Fair-Division Conundrum. *Mathematics Teacher*, *103*(3), p. 172.

Dennis, D. (2000). The Role of Historical Studies in Mathematics and Science Educational Research. In Anthony E. Kelly & Richard A. Lesh (Eds.), *Handbook of Research Design in Mathematics and Science Education* (pp. 799-814). Mahwah, NJ: Lawrence Erlbaum Associates.

Ernest, P. (1991). *The Philosophy of Mathematics Education*. Bristol, PA: Falmer Press.

Harel, G. (1998). Two Dual Assertions: The First on Learning and the Second on Teaching (or Vice-Versa). *The American Mathematical Monthly*, *105*(6), 497-507.

Lenoir, T. (1979). Descartes and the Geometrization of Thought: The Methodological Background of Descartes' Geometry. *Historia Mathematica*, *6*(4), 355-379.

Lesser, L. (2010). Equity and the Increasingly Diverse Tertiary Student Population: Challenges and Opportunities in Statistics Education. In C. Reading (Ed.), *Data and Context in Statistics Education: Towards an Evidence-Based Society. Proceedings of the Eighth International Conference on Teaching Statistics*. Voorburg, The Netherlands: International Statistical Institute. http://www.stat.auckland.ac.nz/~iase/publications.php

Lesser, L. (2009). Equity, Social Justice, and the Mission of TODOS: Connections and Motivations. *Teaching for Excellence and Equity in Mathematics*, *1*(1), 22-27.

Lesser, L. (2007). Critical Values and Transforming Data: Teaching Statistics with Social Justice. *Journal of Statistics Education, 15*(1), 1-21. http://www.amstat.org/publications/jse/v15n1/lesser.pdf

Lesser, L. M., & Winsor, M. S. (2009). English Language Learners in Introductory Statistics: Lessons Learned from an Exploratory Case Study of Two Pre-Service Teachers. *Statistics Education Research Journal*, *8*(2), 5-32.
http://www.stat.auckland.ac.nz/~iase/serj/SERJ8(2)_Lesser_Winsor.pdf

Lim, K. H. (2009). Provoking Intellectual Need. *Mathematics Teaching in the Middle School*, *15*(2), 92-99.

Makar, K. M. (2004). Developing Statistical Inquiry: Prospective Secondary Math and Science Teachers' Investigations of Equity and Fairness through Analysis of Accountability Data. Doctoral dissertation, University of Texas at Austin.
http://www.stat.auckland.ac.nz/~iase/publications/dissertations/04.Makar.Dissertation.pdf

Martin, M. A. (2003). 'It's Like…You Know': The Use of Analogies and Heuristics in Teaching Introductory Statistical Methods. *Journal of Statistics Education*, *11*(2).
http://www.amstat.org/publications/jse/v11n2/martin.html

Mevarech, Z. R. (1983). A Deep Structure Model of Students' Statistical Misconceptions. *Educational Studies in Mathematics*, *14*(4), 415-429.

PsychWiki (2010). Conceptions of Justice/Fairness Scale.
http://www.psychwiki.com/wiki/Conceptions_of_Justice/Fairness_Scale. Accessed on September 7, 2010.

Shaughnessy, J. M. (1977). Misconceptions of Probability: An Experiment with a Small-Group, Activity-Based, Model Building Approach to Introductory Probability at the College Level. *Educational Studies in Mathematics*, *8*(3), 295–316.

Shaughnessy, J. M. (1992). Research in Probability and Statistics: Reflections and Directions. In Douglas A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 465-494). New York: Macmillan.

Shaughnessy, J. M. (2007). Research on Statistics Learning and Reasoning. In Frank K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 957-1009). Charlotte, NC: Information Age Publishing.

Sowell, T. (2010, August 14). Young Graduate Ill-Informed on Diversity. *El Paso Times*, *130*(220), p. 7B.

Stern-Dunyak, A. (1993). Culture Clash: Change and Technological Evolution Coming to the Statistics Classroom. *Amstat News*, No. 202, 13–14.

Svard, A. (2010). Simulating the Risk without Gambling: Can Student Conceptions Generate Critical Thinking about Probability? In C. Reading (Ed.), *Data and Context in Statistics Education: Towards an Evidence-Based Society. Proceedings of the Eighth International Conference on Teaching Statistics*. Voorburg, The Netherlands: International Statistical Institute. http://www.stat.auckland.ac.nz/~iase/publications.php

Thorkildsen, T. (1989). Justice in the Classroom: The Student's View. *Child Development*, *60*(2), 323-334.

Vogt, W. P. (2007). *Quantitative Research Methods for Professionals*. Boston: Pearson.