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Mathematical lyrics: Noteworthy endeavors in education

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Abstract

Mathematical lyrics are song lyrics connected to, or inspired by, mathematics or statistics. This paper explores various types of mathematical lyrics and their roles in mathematics education. In particular, the paper contains many examples of my own lyrics as well as an extensive bibliography of lyrics composed by others. It also provides resources and strategies for creating such lyrics and for using them in an educational setting.

Keywords: mathematical lyrics; statistical lyrics; song; songwriting; mathematical pedagogy

AMS Subject Classifications: 00A65; 97D40; 97F90; 97A99; 97M80; 97C99

1. Introduction

Popular among students of all ages, songs with lyrics can be a valuable vehicle for learning and engagement. Many early examples of the use of mathematical lyrics in educational settings, such as the ABC-TV series *Schoolhouse Rock!* [16], involve elementary school mathematics. However, the use of lyrics as pedagogical tools in college courses across a number of other disciplines [32,46,49,56,62], and several pedagogical experiments in mathematical sciences [34,35,61], indicate a potential for effective use of lyrics to enhance students' learning in college-level mathematics and statistics classes.

Examples of mathematical terms in popular songs include: Möbius strip [19], infinity [28], fractals [20] and others [34,35]. Some lyrics illustrate mathematical concepts without using mathematical terminology. For example, in [2] I illustrated the phenomenon of "regression to the mean" with the following excerpt from "Attractive Stupid People" by Christine Lavin, Ervin Drake, and Gene Weingarten [31].

> but the problem is the kids
> won't look as good as mom or dad
> and they're always slightly smarter
> which drives their pretty parents mad.

Sometimes the connections between lyrics and mathematics are not about content, but about structure. For example, the American rock band Tool sings its "Lateralus" lyric with syllables in groups

whose sizes are successive ascending or descending Fibonacci numbers [59]. The rarity of mathematics in popular song lyrics has motivated many math instructors to write their own mathematical lyrics, such as Tom Lehrer [63], Klein Four Group [29], Steve Sodergren [55], The Fifth Moment [1], Marc Gutman [22], Dane Camp [7], and a math parody band aptly named the Derivatives [4]. While songs can build community at conferences for mathematicians or statisticians (see, for example, [50]), such songs may focus more on an organization and its personalities, while the focus of this article is on songs with content more readily useful in a classroom.

Lyrics and poetry certainly have substantial overlap (e.g., imagery, rhythm, rhyme) and an Internet search unsurprisingly yields many songs inspired by or based on poems. And many lyrics unaccompanied by music can read like fine poetry. However, lyrics and poetry also differ. For example, contemporary poems rarely rely on formal patterned end rhyme, but lyrics generally do. Line breaks add a meaningful dimension to poetry, while melody does this for lyrics. Words and sentences in lyrics tend to be shorter and simpler, and Davis [13, p. 7] elaborates:

> The listener, unlike the reader, gets no footnotes and must understand the lyric as it's being performed. One confusing line or inaudible word will derail the listener's attention….Most important, a lyric is designed to be sung. Its writer…must choose words that roll off the tongue and soar on high notes.

Lyrics may differ even from poetry that is rhythmic and rhymed because a line with non-conversational word order (e.g., other than subject-verb-object) or a homonym can work fine in a written poem, but be hard to follow when someone sings. Interplay with melody gives lyricists additional constraints and opportunities, including: having lyric content match the mood of the music, using grammatical phrasing that aligns with the musical pauses, and placing a lyric's more important words where there are end rhymes, long notes, downbeats, or repetition (e.g., in a chorus). Melody can also reinforce content literally. For example, a science song about amino acids intentionally used high notes when singing the names of amino acids that raise the pH of a solvent and low notes when singing amino acids that lower the pH [12]. Melodic contours could similarly indicate positive/negative or increasing/decreasing characteristics of a mathematical phenomenon. Continuity may be conjured by legato notes (which need lyric syllables ending with vowels or liquid or nasal consonants).

This paper explores various types of mathematical lyrics and their role in mathematics education. Section 2 provides resources and strategies for creating such lyrics. Section 3 discusses the variety of roles mathematical lyrics can play in the mathematics or statistics classroom. Section 4 provides a specific example for the use of mathematical lyrics in an educational setting. The paper contains many examples of my lyrics as well as references to lyrics composed by others. Lyrics and sound-files for some of my songs appear in [9,43].

2. Strategies for writing mathematical lyrics for classroom use

Ideas for songs can be triggered by a particular mathematical term that appears as, or is a homonym of, a word in the title of a familiar song suitable to parody. For example, "Domain and Range" (parodying "Home on the Range") [34] or "American Pi" (parodying "American Pie") [43]. Occasionally, a series of words trigger the same response, such as Ricky Martin's #1 hit "Livin' La Vida Loca" that turned into [43]:

***from* "Findin' Extrema Local"**
by Lawrence Mark Lesser

She'll take out her good pencil and she'll draw a number line,

She'll note where f prime's zero or where f prime's undefined,
And where it changes sign:
Plus then minus means maximum that is local;
Minus then plus means minimum that is local.
This point is critical, so c'mon, let's get vocal!
When f prime swaps sign, findin' extrema local…

Mathematical lyrics can also be generated by conceptual textbook exercises (for example, "From a Distance" [65] was written in response to a college algebra textbook exercise about the behavior of polynomials as determined by their highest degree term), or by issues in the teaching and learning of mathematics (see Section 4 for the motivation for writing "Mean" [9]).

If writing math songs from scratch, that is, composing both original music and lyrics, seems daunting, there are easier entry points. A rap or chant [25,34,58] is easier to write since it uses mainly rhythm and rhyme, and not melody or harmony. While some urban science instructors have used hip-hop and rap to engage students authentically and make concrete connections to curriculum [17], each instructor should consider the particulars of one's identity, comfort zone and students' culture when performing a rap song, especially with a persona. A rap need not even be song-length; it can be a short snippet such as "turn, slide, flip, and glide", to recall the four rigid motions using less formal names for rotation, translation, reflection, and glide reflection.

The next level of difficulty is to modify the lyric of an existing song, utilizing its rhythmic or rhyming structures and its familiar melody and style. While it may be natural to work with songs you grew up on, students have increasingly varied and recent musical reference points, and so it also helps to parody hits by newer pop artists. For maximum classroom effectiveness, it is best to parody songs that reached a high Billboard chart position and are not unduly coarse (lest students recall the original song's vulgarity) or sacred (e.g., a hymn). Effectiveness improves when polling students (in the first week of class) on their favorite artists.

Because class time is precious, it is preferable to streamline songs rather than indulge in a parodied song's extended introductions, solos, or a large number of verses and choruses. For example, Kenny Rogers' "The Gambler" has three verses before the first chorus and ends with three repetitions of the chorus, but the same-titled parody in [41] has only two and one, respectively. This approach suffices to evoke the original song, while focusing more on mathematical content than on musicianship. Beside parodies, another type of lyric-writing to existing notes may use a mathematically-determined melody such as coordinates of points in a fractal image, or the digits of a constant such as **π,** as done, for example, in [47,65].

Just as poems have form (e.g., sonnet, sestina, haiku), so do songs [13,57], and those ready to write a song from scratch should note that some song forms are easier than others. The easiest non-parody song to write may be a 12-bar blues (e.g., "Statistician's Blues" [9], whose last verse is below) consisting of three-line verses where the first two lines are lyrically identical (modulo slight variation in phrasing or interjected words). In other words, each verse is simply a single (possibly imperfectly) rhymed couplet where the first line is repeated.

***from* "Statistician's BLUEs"**
by Lawrence Mark Lesser

She was my significant other – significant at point-oh-three,
She was my significant other – significant at point-oh-three,
But alpha get her soon, as sample as can be!

Song forms are often classified by their pattern of sections, where each section has its own distinct lyrical and musical structure. For example, a song consisting of three musically-identical verses or

sections might be described as AAA, and inserting before the last verse a musically and lyrically contrasting section (often called a "bridge") would turn the song into AABA. At the 2013 XOXO Festival (http://www.youtube.com/watch?v=mV3GWKERCOM), Vi Hart performed a song called "AABA" that had fractal form because the overall song had AABA form and there were also AABA forms within each "A section"! A song with a repeated contrasting section (e.g., ABAB) is often described as having verse/chorus form, and one example in the next section is "Pi Will Go On."

For all types of lyric-writing, there are tools (e.g., http://rhymezone.com, http://rhymer.com, http://bryantmcgill.com/books/rhyming-dictionary) to help one find more interesting or singable words without derailing real-time creativity. Problem solving involved in writing lyrics parallels problem solving in mathematics. As a thought exercise, try to guess whether this interviewee is a mathematician or a songwriter (before checking the answer [14, p. 70]):

> The more technique you have, the more you are familiar with problem-solving, you recognize variations on problems that you've solved before….Over the years, you pick up hundreds of little solutions to problems. The typical problems, everyone learns to solve in the early years….But with the atypical problems, experience and technique become invaluable.

When a song's melody emerges before the words, songwriters often sing a "dummy lyric" that they later revise. The Beatles' classic "Yesterday" ("Yesterday, all my troubles seemed so far away…") initially had very different content over its syllabic pattern: "Scrambled eggs. Oh, my baby how I love your legs…" [51]. Holding one aspect (conceptual, thematic or structural) of a lyric constant while changing another may conjure mathematics, such as the calculation of a partial derivative.

3. Roles of mathematical lyrics in the mathematics classroom

Song lyrics may be used in the mathematics classroom in a variety of situations. For example, instructors may use a lyric they happen to have available about the topic taught that day, playing the song and then posing a follow-up question or activity. It is also possible to simply have a song playing as students arrive to class or have students listen to songs as homework, thus helping set a tone without even taking class time. Instructors may assign the viewing or writing of a lyric as an out-of-class required or optional assignment. And, of course, instructors may bring in lyrics to accompany a larger unit exploring connections between mathematics and music [54]. Because the way a lyric is integrated into a lesson depends in part on the lyric's intended role(s), we now offer examples of these roles.

Aiding Recall: There are many mathematics mnemonics. Some are in the form of an acronym such as FOIL ("first, outer, inner, last"), which helps account for all four product-pairs when multiplying two binomials: $(a + b)\times(c + d) = ac + ad + bc + bd$. A mnemonic in the form of a song, however, has additional dimensions— some borrowed from the poetry of its lyrics, some from its music, and some from their interaction. Imagery, rhyme, metaphor, rhythm, melody, emotion, humor and more, combine memorably to encode and retrieve information, and thus free students' attention for the task of developing a deeper understanding of the material [5,64].

For example, "Circle Song" [37] (below) uses the tune that helps English-speaking children learn the alphabet, to help students recall, or even just distinguish between (since some students remember both formulas, but not which one is which), the area and circumference formulas for a circle.

"Circle Song"
by Lawrence Mark Lesser

Take your finger 'round the jar:
Circumf'rence equals $2\pi r$!

For area, you multiply
r squared by that number $\boldsymbol{\pi}$**.**
Twinkle, twinkle, you're a star:
Knowing math will take you far!

The song's 20 seconds are packed. The first couplet, in addition to providing a concrete example of an object with a circular part, has end rhyme that forces students to realize that the missing letter must be *r*, not *d* (thus helping them recall the correct formula later). The third couplet concludes the song with a math-positive affirmation. The second couplet helps them recall another formula and reminds them that $\boldsymbol{\pi}$ is just a number (though represented by a letter) and its attention on an "*r* square" (i.e., a square whose side has the length of the radius) gives non-calculus intuition into the plausibility of the area formula [18]. The song may be accompanied by a visual demonstration. Have students draw two perpendicular diameters in a circle of radius *r* and then connect the diameter endpoints to form the inscribed square. The inscribed square is divided into 4 triangles that can be assembled into two *r*-by-*r* squares, thus making an area of $2r^2$. Drawing the circumscribed square (whose sides are tangent to the circle at the diameter endpoints) yields a square divided (by the diameters) into four *r*-by-*r* squares, thus making an area of $4r^2$. Since the circle is bounded by inscribed and circumscribed squares, its area must be a number between 2 and 4 multiplied by the square of the radius, and $\boldsymbol{\pi}$ certainly is a number in that window.

Examples of numerous other lyrics that help recall (of procedures, properties, definitions, digits of pi, etc.) may be found in [4,9,34,43].

Introducing concepts or terms: An example of a lyric introducing a concept is "Hotel Infinity", that appears in, for example, [20]. "Hotel Infinity" parodies the surreal hotel-themed Eagles' hit to present David Hilbert's paradox of the Grand Hotel that highlights the counterintuitive properties of infinity. The potential classroom usefulness of this lyric is underscored by the fact that the paradox it presents has been used by researchers [48] to assess students' understanding of infinity. Below is an excerpt from this lyric:

***from* "Hotel Infinity"**
by Lawrence Mark Lesser

On a dark desert highway—not much scenery
Except this long hotel stretchin' far as I could see.
Neon sign in front read "No Vacancy",
But it was late and I was tired, so I went inside to plea.

The clerk said, "No problem. Here's what can be done—
We'll move those in a room to the next higher one.
That will free up the first room and that's where you can stay."
I tried understanding that as I heard him say:

CHORUS: "Welcome to the hotel called Infinity—
Where every room is full (every room is full)
Yet there's room for more.
Yeah, plenty of room at the hotel called Infinity—
Move 'em down the floor (move 'em down the floor)
To make room for more."

Examples of other lyrics introducing mathematical concepts or terms may be found in [9,43].

Reinforcing mathematical thinking processes: Mathematical songs on processes can focus on particular proof techniques (e.g., [6]), the need to check one's work [36], or the process of problem

solving. As an example of the latter, George Pólya's four-step heuristic model [53] morphed into the chorus (see below) of what became "50 Ways to Work a Problem" [34], a parody of Paul Simon's #1 hit "Fifty Ways to Leave Your Lover", a song that others have parodied to teach different mathematics topics [27,52].

***from* "50 Ways to Work a Problem"**
by Lawrence Mark Lesser

Just say what's known, Joan, and what's to find, Caroline.
What rings a bell, Nell, to what we've done?
Make a plan, Stan, follow it through, Sue,
Try to extend, Ken, and we'll be done!

Connecting to history: Songs about the history of mathematics (or math education) may fit especially well in math history and math for liberal arts courses. This example of a pi history song [42, 43] parodies "My Heart Will Go On" from the hugely popular movie *Titanic*:

"Pi Will Go On"
by Lawrence Mark Lesser

Ev'ry time I circle, I see you, I feel you—
That is how I know you go on:
Trillions of digits, no pattern to show us,
You have come to show you go on.
CHORUS: Find π — however we try,
We will see that π does go on.
Before, one guy called it 4,
But I know in my heart that π does go on and on.
22/7 might seem more pleasant,
But π never ends or repeats.
π is a ratio, but never a fraction:
This number really is neat!
CHORUS: Find π — however we try,
We will see that π does go on.
Arctan — however you can,
But Lambert showed that π will go on and on.

This song can be accompanied by a discussion that unpacks the embedded content to ensure students understand, for example, that "trillions of digits" refers to the number of π's (infinite number of) digits that are known so far and that "no pattern to show us" not only means that the decimal expansion of π does not begin to repeat the same finite sequence of digits, but also invokes the likely (but not yet proven) trait that each of the digits 0-9 occur in π with equal frequency. Eighteenth-century Swiss mathematician Johann Lambert proved that if $\tan(x)$ is rational, then x is irrational, and since $\tan(\pi/4)$ equals the rational number 1, then $\pi/4$ (and therefore π) cannot be rational [30]. The word "arctan" in the lyric motivates investigating how series expansions of functions such as the arctangent function have been historically used to obtain increasingly better approximations of π. Instead of simply presenting these connections, some instructors may prefer to ask students to track them down. Especially interesting is to steer students towards finding the bill a doctor introduced to the Indiana legislature requesting that π be legally made equal to 4 [23]. Another math history song appears in [38].

Connection to the real-world: "The Gambler" (see [41] or http://www.youtube.com/watch?v=PxGRghzr5zo) parodies Kenny Rogers' same-titled signature song and #1 country hit for the purpose of educating about playing the lottery. This song may be used in both formal and informal statistics classes to motivate or punctuate discussions about randomness, probability, and expected value. It can act as a good starting point for a mathematical exploration [40] that questions whether marketed so-called "secrets to win the lottery" (such as tracking past numbers drawn) have any effect on the probability of winning or on the expected value of one's winnings.

***from* "The Gambler"**
by Lawrence Mark Lesser

You track those weekly draws, you say ya got a system—
You call some numbers "hot", you deem others "due";
But I insist, they each have the same chance—
If you're gonna play the game, boy, ya gotta know what's true!

CHORUS: You gotta know when you pick 'em what's superstition,
Know what is strategy and know when there's none!
You never try to learn this at the 7-11:
Take the time right now for learnin' when the singin's done!

Humanizing mathematics: Some songs aim less to convey specific content, and more to let the emotional unifying vehicle of music offer inspiration and motivation to engage an often dreaded subject. Such humanizing songs can include anthems to encourage or celebrate student success [39] or simply use mathematics playfully [8,33] to help reduce the math anxiety students too often feel. Vi Hart sings "I'll Be Here" [24] from the point of view of mathematics itself to someone with ambivalence towards it, and thus her song could be used to discuss attitudes toward mathematics. In general, math-positive songs may help counteract negativity in our popular culture, as reflected by Jimmy Buffet's song "Math Suks" [34].

4. Using lyrics in a class

In this section we will explore the use of lyrics in a particular course—a statistics literacy course that I have taught 15 times over the last decade at The University of Texas at El Paso, most recently in spring 2013. The course enrolls primarily pre-service elementary and middle school teachers. One of the songs I performed live in this course was "Mean" (below), a parody of the same-titled song that was the Grammy-winning country song of 2012 for Taylor Swift and a recording is freely available at [9]. While I hoped that the song would boost engagement (since the original song was performed by one of their favorite current young pop artists), my main reason for writing and performing the song was to address a specific content weakness that I noticed when previously teaching this course and also read about in statistics education literature [21].

"Mean"
by Lawrence Mark Lesser

You— with your words like mean and mode and symbols that you use against me.
μ stands for a population mean and $\bar{x}$ for the sample.
Who knows what symbol to use for other measures of data location.
You, pickin' each time the mean.
But it can get pulled off by just one single point,
And you don't know what you don't know....

CHORUS: Sometimes data have a real asymmetry
Or values that are really quite extreme.
In those cases, better use the median
If the middle is what you need— that's not gonna be the mean!

That won't be the mean with outliers, just forget it,
Do you see just what I mean and mean and mean and mean?
(Repeat Chorus)

I introduced this mathematical lyric after covering the definitions, calculations, and some properties of measures of centers of datasets. In particular, we discussed when one measure of a center is more appropriate than another (a topic covered in section 7.5 of our textbook [60]). Two possible ways of measuring the center of the observations in a dataset are the mean and the median. The mean is the quantity obtained by summing up the values of all the observations in the dataset and then dividing by the total number of observations, while the median is the middle value of a set of observations ordered from smallest to largest. This lyric was used to reinforce memorably that the mean is generally an inappropriate measure for the center of the dataset when its observations are highly skewed. The reason for this is that a few unusually high or unusually low observations in the dataset would have undue influence when calculating the sample mean. In conjunction with this song, I facilitated several class discussions about concrete datasets. For example, one of the datasets we discussed consisted of observations of household incomes whose distribution was skewed to the right by the presence of unusually high incomes of CEOs, etc. Students readily agreed that it would be misleading to use the mean to calculate a "typical" household income (that is, a middle-of-the-pack income), since the mean would be significantly higher for this type of distribution. Indeed, a search for household incomes on the Census Bureau website yielded a report [15, p. 33] listing the 2012 median household income as $51,017, while the mean was 40% higher: $71,274. This topic was reinforced with homework assignments and exam questions, but in a way that was not tied explicitly to the song.

In spring 2013, a researcher in mathematics education used my class to conduct a qualitative case study [45] of pedagogical strategies for engagement. To this end, the researcher conducted non-participant observation of five class meetings and confidential individual interviews with six students. All of the interviews and most of the class observations were transcribed. At the 2/25/13 class meeting, during which I played "Mean", the researcher recorded these observations in his field notes:

> the professor nonchalantly says that 'Taylor Swift was going to come by and sing something, but she had a concert to go to and asked me to do it instead.' The professor takes out his guitar and the whole class laughs and seems excited with anticipation. He then proceeds to sing 'Mean'. All students are smiling, focused on the professor, with several recording him with their mobile devices.

Before I began the song, I projected the lyric on the screen and invited students to feel free to sing along, especially during the singing of the chorus. Although most students did not sing out loud, all seemed engaged and I saw many quietly mouthing the words and smiling at the moments the lyric most evoked the parodied song. In general, students are more willing to join in with "fill-in-the-blank" lyrics in which they have to guess and sing only the last word of each couplet (such as "Circle Song"), and even more willing to produce a rhythm to accompany a mathematical rap. The subsequent interviews the qualitative researcher conducted revealed that students found "Mean" and the other songs I shared with the class to reduce in-class stress and increase attention and engagement [45]. While a rich qualitative description of the use of song lyrics in a mathematical sciences classroom is extremely valuable, it is also helpful to collect a large sample of quantitative data on learning outcomes associated with the exposure to song and this is discussed in the next section.

5. Concluding remarks

As part of a grant (see Acknowledgments section), data collected from 147 students in a fall 2013 randomized experiment at a university and a community college (with different and diverse student populations) were analyzed to see if students exposed to songs inserted into otherwise conventional self-contained content in their course management system perform better on related multiple-choice items embedded on midterm exams than students who received the online content without the song inserts. This structure not only allowed all other aspects of the student experience to be identical, but also removed effects of instructor implementation or talent. Overall, for students who were randomly selected to always be exposed to lessons with an inserted related song, these embedded exam questions were answered correctly an average of 50.0% of the time; for students who were randomly selected to always be exposed to the same lessons without songs, questions were answered correctly 42.3% of the time—a difference (50.0% - 42.3% = 7.7%) with statistical observed significance $p \approx 0.04$, as well as practical significance for the students of 2/3 of a letter grade. Separately, for each of the six statistics songs in the study, the difference in embedded exam question performance (given as percentage of correct answers) always favored the song-exposed group of students and was as high as 73.8% - 60.2% = 13.6% for one particular song. The same study [44] also investigated whether the students exposed to the song inserts had better attitudes towards statistics or lower statistics anxiety, but no significant differences were found. Further research is merited, as is the establishment of a curated searchable database of mathematics songs as a counterpart to science and statistics song collections [9,11].

Just as pedagogical experiments in poetry writing have occurred in mathematics classes (see concluding remarks of [20]), educators can explore how to involve their students in the writing of mathematical lyrics. For example, the quadratic formula has been set to the tune of many songs and many mathematical topics have been covered using the lyric "100 Bottles of Beer" [3]. Instructors can find an appropriate method and level for students to analyze or create lyrics in their courses [10,26].

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