

2017 NCTM Annual Meeting and Exposition

# Changing the Rhythm of Math Class: Using Educational Songs to Cultivate Learning and Community



**Larry Lesser** (The University of Texas at El Paso)

joint work with Dennis Pearl (Pennsylvania State University), John Weber (Perimeter College at Georgia State University), and Dominic Dousa & Steve Haddad (UTEP)



<http://www.math.utep.edu/Faculty/lesser/Fun.html>

(or you can Google my “Mathemusician” page)

supported in part by NSF grant **Project**



**Student-Made Interactive Learning with Educational Songs** (for introductory statistics)

PSU (1544426); UTEP (1544237); GPC (1544243)



the math of UTEP’s Bhutanese architecture was in Sept. 2008 *Mathematics Teacher!*

for lots of math & music resources, lyrics, MP3s, etc.,  
just *Google* me!



Larry Lesser

Search

About 8,820,000 results (0.24 seconds)

Web

[Larry Lesser - Mathematical Sciences](#)

[www.math.utep.edu/Faculty/lesser/](http://www.math.utep.edu/Faculty/lesser/)

QUICK BIO: Statistics/math educator **Larry Lesser** has taught university math, statistics and math education courses in Colorado, Georgia, and Texas and since ...

[Math Ed - Statistics - Equity - Study Tips](#)

Images

Maps

Videos

News

Shopping

[Mathemusician.html - University of Texas at El Paso](#)

[www.math.utep.edu/Faculty/lesser/Mathemusician.html](http://www.math.utep.edu/Faculty/lesser/Mathemusician.html)

ABOUT THE MATHEMUSICIAN: To support his mission in increasing awareness and motivation in mathematics, Professor Lesser has tapped and merges two ...

# my math song resource page

just Google: “Larry Lesser” or “Mathemusician”

[www.math.utep.edu/Faculty/lesser/Mathemusician.html](http://www.math.utep.edu/Faculty/lesser/Mathemusician.html)

<a href="#">Resources &amp; Bibliography</a>	<a href="#">SONGS!</a>	<a href="#">Cool Math &amp; Music Quotes</a>
<a href="#">More on Lesser</a>	<a href="#">Presentations &amp; Press</a>	<a href="#">Contact Info</a>



## *The Mathemusician*

“Find, find the value of pi -- starts 3 point 1 4 1 5 9.  
A good ol’ fraction you may hope to define, but the decimal never dies ....”  
-- from Larry Lesser’s award-winning “[American Pi](#)”



**ABOUT THE MATHEMUSICIAN:** To support his mission to motivate learning mathematics/statistics, Professor Lesser integrates two great loves – math and music! An award-winning songwriter in general songwriting contests, Lesser has also won awards for educational songs and song videos in recent national contests ([ASA](#), [National Museum of Mathematics](#), [CAUSE](#), [QL-SIGMAA](#)) and a [video of songs was shown at Bridges 2014](#). His songs have yielded statistically significant learning gains in an [NSF-funded randomized experiment](#), helped

## Changing the Rhythm of Math Class: Using Educational Songs to Cultivate Learning and Community

### General Interest Session

We'll discuss (and demo!) best practices (informed in part by our NSF grants) for creating and using educational songs for grades 8–14 mathematics/statistics. We address perceived barriers and hesitations by sharing low-risk, high bang-for-the-buck strategies, resources, and tips for finding, writing, and using songs aligned to learning objectives.

Lawrence Lesser

University of Texas at El Paso (UTEP), Texas

Dennis Pearl

Pennsylvania State University, University Park

John Weber

Perimeter College at Georgia State University, Clarkston

Henry B. Gonzalez Convention Center, 214C

# OUTLINE

- *Background*
- Addressing Hesitations
- Finding Songs
- Using Songs
- Writing Songs
- Interactiveness
- Q&A

Opening Question  
(and quick detour):

Name a connection  
between math & music!

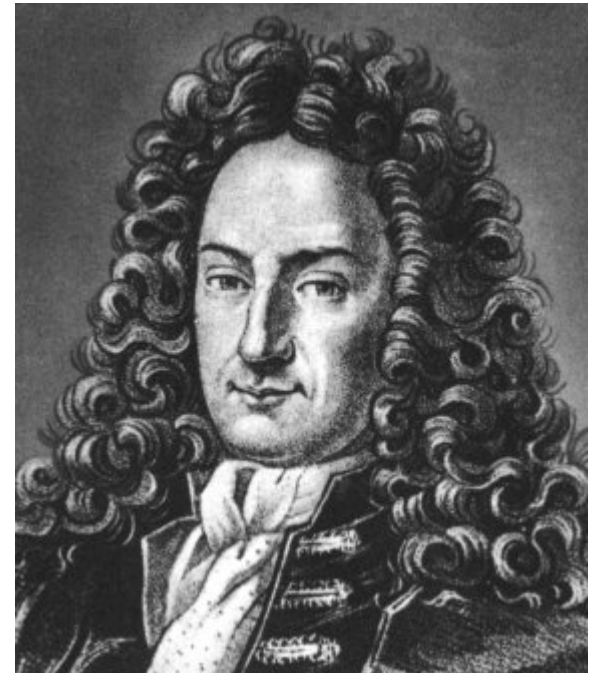
<b>MATH</b>	<b>MUSIC</b>
Ordinal numbers	naming intervals (e.g., fifths)
Geometric shapes	instrument shapes (e.g., triangle)
Geometric transformations	melodic transformations
Least common multiple	rhythm patterns; harmony (from pitches w/ low LCM)
Fractions	time signature; interval is a ratio of frequencies
Arithmetic sequence	overtones (f, 2f, 3f, 4f, ...)
Geometric sequence	chromatic scale (in equal temperament)
Graph (pitch over time)	musical notes on a staff
Sine function	graph of (pure tone) sound wave
Permutations, Probability	“change ringing” of bells; Mozart’s “Musical dice game” (1793)
Statistics	DATA→SOUND: sonification SOUND→DATA: traits of hit songs
Group theory, modular arithmetic	analyze the set of pitches in a scale
Fourier series, partial differential equations	musical sound, acoustics

# sample M&M quote from my webpage

<http://apusepress.booktype.pro/early-readings-in-the-philosophy-of-science/gottfried-wilhelm-leibniz-1646-1716/>

**“Music is the pleasure  
the human soul experiences  
from counting  
without being aware  
that it is counting.”**

-- calculus co-inventor  
Gottfried Leibniz



# Connections with structure of music:

demos with acoustics, music theory, etc.

(CCSSM Mathematical Practice standard #7: “look for and make use of **structure**”)

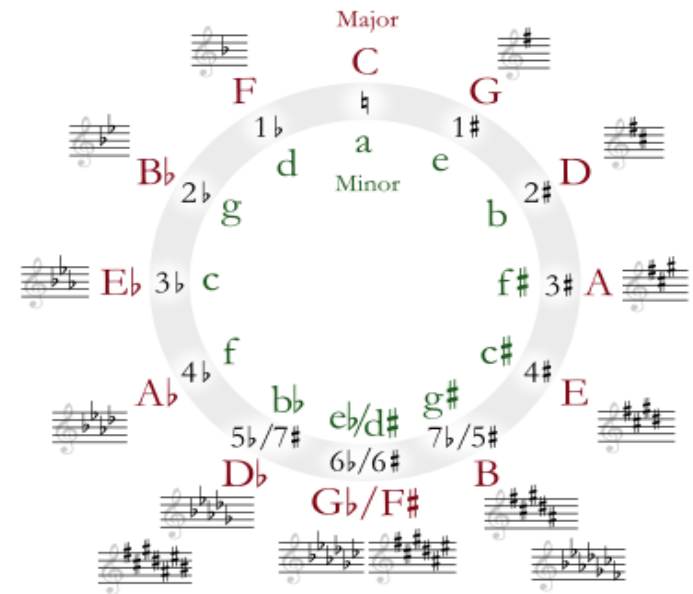
e.g., see Sept. 2009

*Mathematics Teacher* for

how **group theory**

and **geometric transformations**

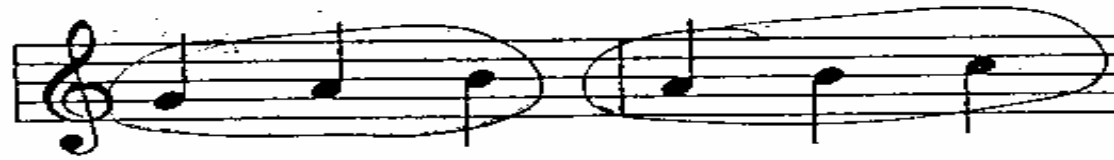
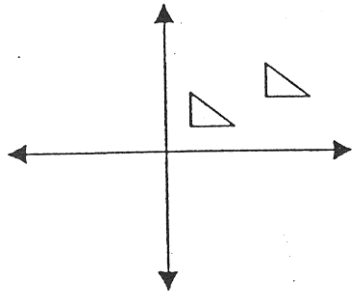
connect to music



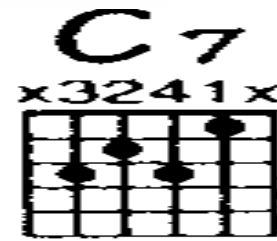
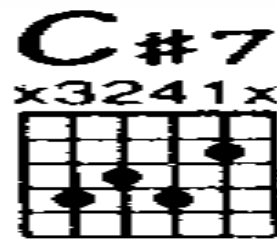
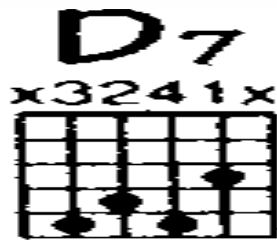


# Translations

in music:  
transpositions



works with chord shapes, too

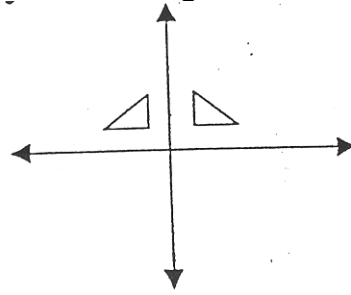


# capo



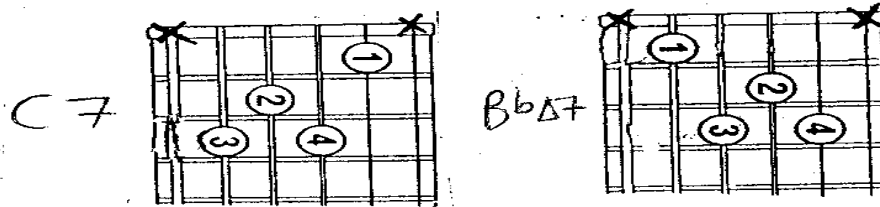
# Reflections

in music:  
retrogression



notes on a staff:  
a graph of pitch over time

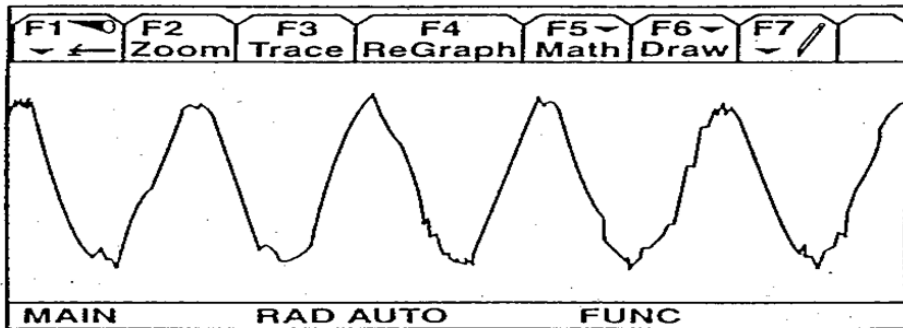
works with chord shapes, too



as a "lefty",  
I did this in my head

# classroom demos

mike → CBL → GC:  
hit a tuning fork, blow  
across soda bottle, etc.



pick a frequency to graph  
and play

The WolframAlpha interface for the query "play 440 Hz sine wave". The input field contains "play 440 Hz sine wave". The "Input interpretation" section shows "pure tone" and "frequency 440 Hz (hertz)". The "Amplitude function" section shows the equation  $\sin(880 \pi t)$  (t in seconds). The "Waveform" section displays a blue sine wave graph with the x-axis labeled "time (seconds)" and tick marks at 0.000, 0.002, 0.004, 0.006, and 0.008. A "Play" button is visible in the top right corner of the waveform section.

but today, our focus is

**SONG**

(i.e., including lyrics)

# math in lyrics of “**regular** songs” ...

- “One is the loneliest number” – Three Dog Night
- “if I could count infinity” – Christine Kane
- “100% chance of rain” – Gary Morris
- “reduce me to the mean” – Richard Shindell
- “random sample, hold the one you need” – Rush
- “at the edges of the bell-shaped curve” – David Wilcox
- “it’s like a Möbius strip” – Nelly Furtado

OR concept without terminology: *regression to the mean* in Christine Lavin’s “Attractive Stupid People”

**know others? email me!**

Raise your hand if....

no song has ever

helped you learn something

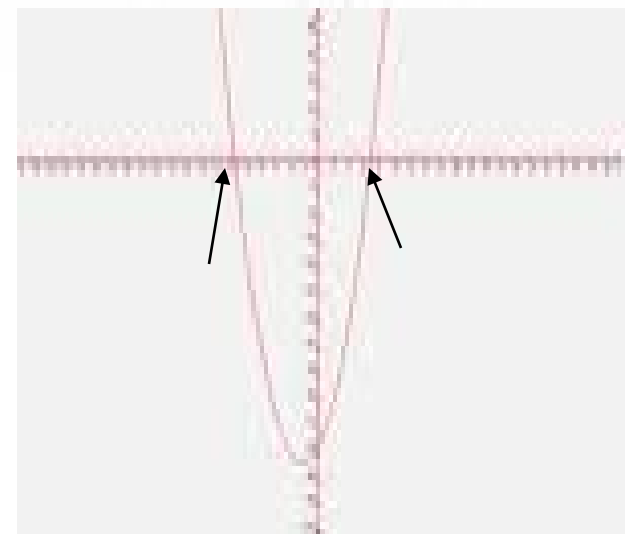
# song helped many learn:

- “**The Alphabet Song**” to learn ABCs
- Ray Charles’ “**Fifty Nifty United States**” to learn 50 U.S. states (in alphabetical order!)
- about 60 Schoolhouse Rock (on ABC) songs spanned **Multiplication** (by 2,...,12), Grammar (e.g., “Conjunction Junction”), America, Science, Money, Earth

# QUIZ: Which tune(s) has/have been used for the quadratic formula?

- a) Pop Goes the Weasel
- b) Frère Jacques
- c) Battle Hymn of the Republic
- d) Amazing Grace
- e) 'Gilligan's Island' theme
- f) Macarena
- g) A school 'fight song'
- h) All of the above

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$





# my M&M trajectory

- playing guitar, began songwriting in college
- brought music & song into my college math teaching in 1990s
- then....



# Math teachers flunk Buffett tune

The National Council of Teachers of Mathematics is not happy about Jimmy Buffett's new song, *Math Suks*, found on his just-released *Beach House on the Moon* album.

"It's a very, very negative song," Michigan State University professor and council president Glenda Lappan says. "It ends with 13 'math suks' in a row."

Adds Lappan: "This is a great disappointment. I'm a serious Jimmy Buffett fan. I have tapes in my car."

The Reston, Va.-based council issued a statement condemning the tune, saying: "In a world dominated by technology and computer-driven devices, today's students need to be inspired more than ever to learn and understand mathematics."

Lappan says, "In our culture, we tend to look for excuses for



**Buffett:** His 'Math Suks' lacks a 'positive spin,' teachers say.

exchange.

What does the council expect Buffett to do?

It will ask him that, "as a recording artist and icon, he say some things that would put a more positive spin on mathe-

# "Math Suks"

July/August 1999

NCTM News Bulletin

## A Conversation with Jimmy Buffett Shows Math Doesn't "Suk" After All

For educators, Jimmy Buffett's new song posed a dilemma. Is "Math Suks" a cause for angst, introspection, or just comic relief? For Buffett, an educator himself, it's clearly the latter. "I apologize—I wasn't trying to attack mathematics education," says Buffett, a popular singer and songwriter as well as a best-selling author, in an interview with NCTM's *News Bulletin* editor. Rather, the song provided Buffett, known for his lighthearted sense of humor, a chance to parody what seemed to be a universal experience—the struggle to learn math.

The title, however, was a little disconcerting to some mathematics teachers, even those with a generous sense of humor and Jimmy Buffett CDs by the stereo. Buffett's song seemingly contradicts the messages NCTM sends about mathematics. Most recently, NCTM's "Do Math ... and You Can Do Anything" public service campaign has been trying to instill in students the realization that mathematics can be a key to the future. But the song includes references to some timeworn stereotypes—such as notions that math is hard, that it's boring, and that women, especially beautiful ones, can't do it. Is "Math Suks" a setback?

# May 2000 *MT* has 1st comprehensive article on song in math class

Lawrence Mark Lesser



## Sum of Songs: Making Mathematics Less Monotone!

Songs in the mathematics classroom  
can be fun and functional; they can supply motiva-

a long-run frequency? What do probabilities mean in weather forecasting, for example, will 100 percent of the geographic area receive rainfall? Is the estimate a statement of history? Is the weather forecast wrong if it does not rain?

Statistical language is also used in such lyrics about resisting mainstream conformity as this one by the rock band Rush (1981):

Everybody got mixed feelings about the function and the form;  
everybody got to deviate from the norm.

Another example is this lyric from singer-songwriter David Wilcox (1999):

# then Autumn 2001 *TS* has 1st article on song in statistics class

**TEACHING STATISTICS**  
An International Journal for Teachers

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## Musical Means: Using Songs in Teaching Statistics

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**KEYWORDS:**

*Teaching;*

*Lyrics;*

*Songs;*

*Hypothesis test;*

*Permutations;*

*Mozart effect.*

*Lawrence M. Lesser*

Armstrong Atlantic State University, Georgia,  
USA.

e-mail: lesserla@mail.armstrong.edu

**Summary**

Songs offer fresh motivations for probability and statistics.

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### ◆ INTRODUCTION ◆

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Teachers are always looking for fresh realms to increase the interest in associated concepts of statistics. Few areas of life appeal to youth as much as music. Simonite (2000) reports on data that were quickly generated by testing university students and staff with questions about pop music from the 1960s and the 1990s. Simonite explains that having 'a ready understanding of the context allowed students to suggest theories explaining variations in scores which were then tested statistically' (pp. 58–9).

Students' ready understanding of (and interest in) the context of songs can be utilized more generally throughout the probability and statistics curriculum. Adding to some of the ideas of Lesser (2000), we present several types of examples, in-

be displayed using a number of representations, including a pie chart, bar graph or pictograph. Primary school teachers may also utilize bottle graphs (in which children cast votes by adding a standardized cup or eyedropper of water into one of several equal-sized clear bottles or cylinders and then examine the final water levels; see Bamberger and Hughes 1995) or glyphs (pictures whose features convey information according to a legend; see Harbaugh 1995).

There is also a wide variety of quantitative data that can be collected and summarized, such as the length of time of songs or the number of words in titles of hit songs. These data also can be displayed using a number of representations, including a histogram, a stem-and-leaf plot, a box-and-whisker plot or (when examining two quantitative variables simultaneously) a scatterplot.

3/31/2002 [Melbourne] Herald Sun

Australia's largest newspaper called my lyrics "bizarrely effective"...

# Sound way to learn maths

A teacher puts complex maths theories to music... and SAMANTHA AMJADALI finds it adds up

Larry Lesser has discovered the secret to fostering a love for mathematics in his students — Kenny Rogers' catchy ballad *The Gambler*.

Dr Lesser, a mathematician with a penchant for song, found his charges couldn't grasp the bamboozling theories in his university courses. So he simply put the complex formulae and equations, to music. Now students flock to his classes — and they're getting better marks.

His students learn to memorise the value of Pi to the tune of Don McLean's *American Pie*, they're taught the features of a graph to Bette Midler's *From A Distance* and, for those interested in the mathematics of random occurrences such as lotteries it's *The Gambler* which they gleefully sing along to.

Dr Lesser said his two-minute maths ditties were generally the only thing a student would recall from a lesson — sometimes the entire semester's course.

"Although music does help

with memorisation, I use it more for motivation," said Lesser, a maths professor at Armstrong Atlantic State University in Savannah, Georgia.

"They're certainly outstanding mnemonic devices to help students learn and recall information, often with more enjoyment or less anxiety."

Dr Lesser chooses rap songs for memorising basic facts, but prefers longer, catchier tunes such as *We Will Graph You*, a reworking of Queen's classic *We Will Rock You*, (lead singer Freddie Mercury is pictured above) for learning fundamental theories such as the procedures needed for graphing a line or parabola.

But Dr Lesser won't whack his precious words into any old song. He chooses his songs lovingly and avoids profane songs and sacred tunes, preferring instead to use chart toppers that his generally teenage students will know.

[www.math.armstrong.edu/faculty/lesser/Mathmusician.html](http://www.math.armstrong.edu/faculty/lesser/Mathmusician.html)

## The method

Freddie Mercury's algo-rhythmic vocals have been "re-purposed" by Larry Lesser, who teaches students to graph a given function using the tune of Mercury's *We Will Rock You*.

The students must chant and clap the song in unison.

Lesser's bizarrely effective rendition is called *We Will Graph You* and goes like this:

You wanna draw a parabola from the general form algebra.

Don't despair, complete the square. The  $x$  of the vertex comes out there, and we will, we will graph you! (chorus twice repeated).

The vertex now is figured out, but does the graph smile or frown?

The number next to the square of  $x$ , gave the sign to make the sketch, now — we will, we will graph you! (twice repeated).

Find and plot any  $x$ -intercepts, the constant  $c$  is the  $y$ -intercept.

A vertical line through the vertex, gives symmetry for your sketch, and we have, we have graphed you! (twice repeated).



# conference edutainment

- for **mathematicians** (e.g., MAA MathFest opening banquet)
- for **statisticians** (e.g., USCOTS banquet)
- for **K-12 teachers** (e.g., opening session of NCTM Regional in Nashville)

7:00 p.m.–8:00 p.m.

1



## Composing Connections: Mathemusician Merges Math and Music in the Music City!

(General Interest) Session

A mathematics education professor, former schoolteacher, and published songwriter will share inspiring, diverse, and accessible connections between math and music that can help increase students' motivation and appreciation for math. His engaging math songs and raps have appeared in many journals and had international mass media coverage.

**Lawrence M. Lesser**

University of Texas at El Paso, El Paso, Texas

205 (Convention Center)



# from my years teaching HS...

- Did a math rap in teaching on my interview!
- Made connections to students and learned to appreciate (and parody) songs from newer artists (John Mayer, then Taylor Swift, Carly Rae Jepsen) or movies (*Titanic*)
- Students inspired math topics

# Example:

a song I wrote to help my  
HS geometry students recall &  
distinguish 2 main **circle** formulas

using the tune that helped us all  
learn the **alphabet!**



# “Circle Song”

lyric © 2004 L. Lesser

you complete each rhyme!

Take your finger ‘round a jar --

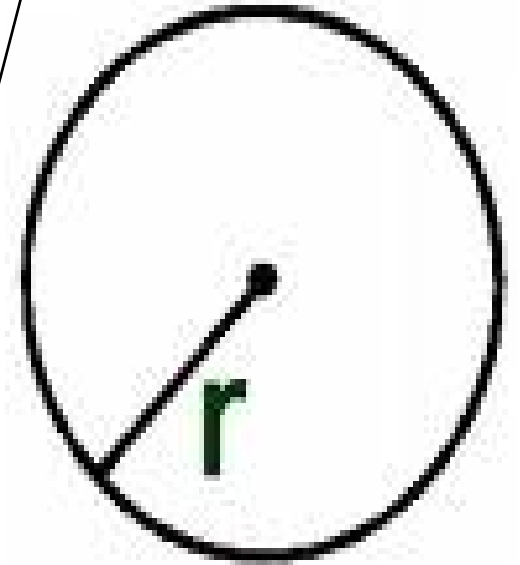
Circumf’rence equals 2 pi \_\_\_\_\_;

For **area**, you multiply

R squared by that number \_\_\_\_\_.

Twinkle, twinkle, you’re a star

Knowing math will take you \_\_\_\_\_!



## Circle Song

by Lawrence Mark Lesser

Take your finger 'round the jar:  
Circumference equals  $2\pi r$ !  
For area, you multiply  
 $r$  squared by that number  $\pi$ .  
Twinkle, twinkle, you're a star:  
Knowing math will take you far!

## Lesser (2014), *JMA*

The **first couplet**, in addition to providing a concrete example of an object with a circular part, has end rhyme that forces students to realize that the missing letter must be  $r$ , not  $d$  (thus helping them recall the correct formula later).

The **third couplet** concludes the song with a math-positive affirmation.

The **second couplet** helps them recall another formula and reminds them that  $\pi$  is just a number (though represented by a letter) and its attention on an ' $r$  square' (i.e., a square whose side has the length of the radius) gives non-calculus intuition into the plausibility of the area formula.

## Flores & Regis (2003), *MTMS*

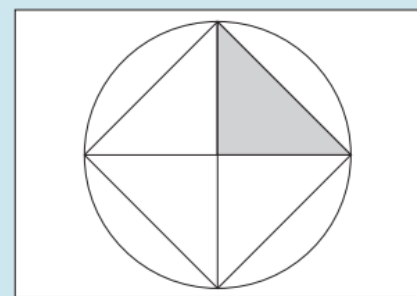


Fig. 2 The shaded triangle is one-half of the radius square.

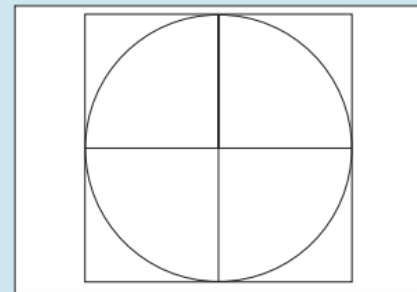


Fig. 3 The area of the circle is less than the area of four radius squares.

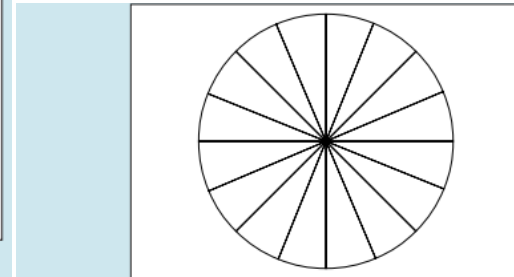


Fig. 9 A circle cut into sixteen slices

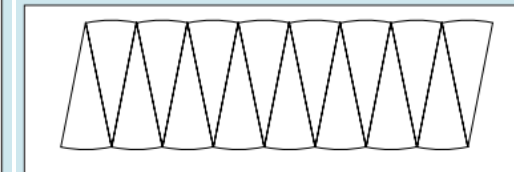


Fig. 10 The slices of the circle rearranged

# math & guitar/monochord connections with K-12 kids

Local PBS kids show (gr. 1-3)  
guest on BB (ep. 29 of season 1; 10/20/2011)



lesson for local 7<sup>th</sup> & 8<sup>th</sup> graders  
(KFOX-TV 5pm news, 2/1/2013)



# 2013 skateboard/**guitar** article in *European J. of Science and Mathematics Education*

- How frequency varies with a string's length, tension, mass
- How frequency ratios sound
- Location patterns of guitar harmonics and frets

## **Scientific skateboarding and mathematical music: edutainment that actively engages middle school students**

**William Robertson<sup>1</sup>, Lawrence M. Lesser<sup>2</sup>**

<sup>1</sup> Department of Teacher Education, The University of Texas at El Paso, El Paso, Texas, USA

<sup>2</sup> Department of Mathematical Sciences, The University of Texas at El Paso, El Paso, Texas, USA

For correspondence: robertson@utep.edu

**Abstract:** Edutainment has recently been a major growing area of education, showing great promise to motivate students with relevant activities. The authors are among innovators who have developed cutting-edge fusions of popular culture and STEM concepts to engage and to motivate middle school students, using vehicles such as music/song and skateboarding. The importance of using relevant and practical methods of instruction and curriculum delivery that build on student interests and increase enjoyment in the learning process are critical at the middle school level, especially in the STEM fields. The use of edutainment in this manner is meant to inspire broader interest in mathematics and science for middle school students and to develop a culture of education that makes learning more accessible to all students. This paper surveys and illustrates the use of such immersive modalities to involve middle school students actively with concepts and suggests further directions for the use of demonstrations and videos in educational settings.

**Keywords:** action, science, mathematics, skateboarding, music.

# in (Holt) *Algebra 2*

## EXAMPLE

6

### Music Application

Frets are small metal bars positioned across the neck of a guitar so that the guitar can produce the notes of a specific scale.

To find the distance a fret should be placed from the bridge, multiply the length of the string by  $2^{-\frac{n}{12}}$ , where  $n$  is the number of notes higher than the string's root note. Where should a fret be placed to produce a G note on the E string (3 notes higher)?

$$64\left(2^{-\frac{n}{12}}\right) = 64\left(2^{-\frac{3}{12}}\right)$$

Use 64 cm for the length of the string, and substitute 3 for  $n$ .

$$= 64\left(2^{-\frac{1}{4}}\right)$$

Simplify.

$$= 64\left(\frac{1}{2^{\frac{1}{4}}}\right)$$

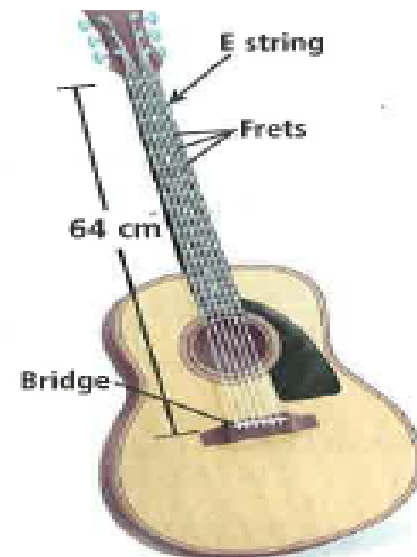
Negative Exponent Property

$$= \frac{64}{2^{\frac{1}{4}}}$$

Simplify.

$$\approx 53.82$$

Use a calculator.





```
64 / (2^(1/4))
53.81737058
```

The fret should be placed about 53.82 cm from the bridge.



6. Where should a fret be placed to produce the E note that is one octave higher on the E string (12 notes higher)?



# 2014 *J. of Mathematics and the Arts* paper



## Journal of Mathematics and the Arts

Special Issue:  
Poetry and Mathematics  
Guest Editor: Sarah Glaz



Taylor & Francis  
Taylor & Francis Group

### Mathematical lyrics: noteworthy endeavours in education

Lawrence M. Lesser\*

Department of Mathematical Sciences, The University of Texas at El Paso, El Paso, TX, USA

(Received 26 January 2014; accepted 29 July 2014)

Mathematical lyrics are song lyrics connected to, or inspired by, mathematics or statistics. This paper explores various types of mathematical lyrics and their roles in mathematics education. In particular, the paper contains many examples of my own lyrics as well as an extensive bibliography of lyrics composed by others. It also provides resources and strategies for creating such lyrics and for using them in an educational setting.

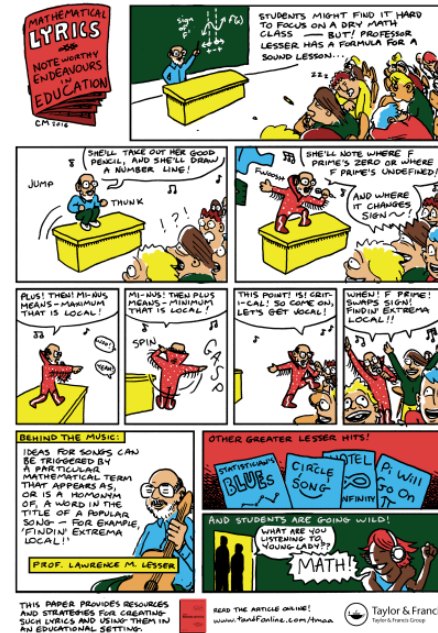
**Keywords:** mathematical lyrics; statistical lyrics; song; songwriting; mathematical pedagogy

**AMS Subject Classifications:** 00A65, 97D40, 97F90, 97A99, 97M80, 97C99

#### 1. Introduction

Popular among students of all ages, songs with lyrics can be a valuable vehicle for learning and engagement. Many early examples of the use of mathematical lyrics in edu-

such as Tom Lehrer [63], Klein Four Group [29], Steve Sodergren [55], The Fifth Moment [1], Marc Gutman [22], Dane Camp [6] and a math parody band aptly named the Derivatives [4]. While songs can build community at



# there are international math song contests!

National Museum of **Mathematics** (MoMath)

- Pi Day of the Century contest – spring 2015
- “Open Set” contest – fall 2015
- “Open Set” contest – spring 2017

Consortium for the Advancement of  
Undergraduate **Statistics** Education (CAUSE)

- A- $\mu$ -sing contest – odd springs 2007-2017

I was a winner in song contests held by the National Museum of Mathematics for “Pi Day of the Century” (3/14/15), fall 2015, spring 2017



### Math Song Wins on Pi Day of the Century

Last Updated on March 16, 2015 at 10:20 am

EL PASO, Texas – Director of the Center for Excellence in Teaching and Learning at The University of Texas at El Paso Larry Lesser, Ph.D., is now a national award-winning songwriter.

In honor of the Pi Day of the Century, since the date March 14, 2015 (3/14/15), is the only date in the 21<sup>st</sup> century that contains the first five digits of the number pi (3.1415), the Museum of Mathematics in New York City sponsored a related worldwide song contest.

The nation’s only math museum attracted more than 100 entries. Lesser entered two pi-themed songs, “American Pi” (a parody of Don McLean’s No. 1 hit “American Pie”) and “Pi Will Go On” (a parody of the Celine Dion hit “My Heart Will Go On” from the movie Titanic). “American Pi” was announced as the winner at 3:14 p.m. ET on Pi Day, March 14, at the museum.

## UTEP professor wins Pi Day of the Century songwriting contest

By Aaron Martinez / El Paso Times

POSTED: 03/14/2015 05:26:19 PM MDT

### Listen to Greatest Lesser Hits

It will only happen once this century and for a math professor at the University of Texas at El Paso, Pi Day 2015 will be a day he will never forget.

Out of more than 100 entries from across the globe, UTEP Mathematical Sciences Department Professor Larry Lesser's was declared the winner of the Museum of Mathematics' Pi Day of the Century songwriting contest on Saturday.

"It was a really hoot to win this contest," said Lesser, who is director of the Center for Excellence in Teaching and Learning. "It was just shocking and such a great honor to win a national, well,





# 2015 *J. of Mathematics Education* paper

Journal of Mathematics Education  
December 2015, Vol. 8, No. 2, pp. 158-168

© Education for All

## **“American Pi”: The Story of a Song about Pi**

**Lawrence M. Lesser**

*The University of Texas at El Paso, USA*

*This paper begins by overviewing motivations and means for using music in the teaching of mathematics – in particular, six roles for the use of song. We then share inspirations and variations for the award-winning song “American Pi” (which parodies a song that topped the charts in the United States, Australia, Canada, and New Zealand), followed by overviewing several options for implementation in the mathematics classroom, especially the high school classroom. It is hoped that focusing on characteristics and trajectory of one particular mathematics song may help yield a framework or context for examining, using, and writing other mathematics songs.*

**Key words:** Pi, lyric, song, humanistic mathematics, mathematics history.

### **Motivations**

There are many ways music can be used to motivate or facilitate the learning of mathematics. Robertson and Lesser (2013) include many references

*to recall (6 sig. figures of)  $\pi$*

Chorus of “**American Pi**” (lyric © L. Lesser):

Find, find the value of pi, starts 3 point 1 4 1 5 9

A good ol’ fraction you may hope to define

But the decimal never dies,

The decimal never dies....

*find this song on my “Pi Day” webpage!*

like history teachers use  
Billy Joel's "We didn't start the fire"

In preparation for PI DAY,  
I share the lyrics of "American Pi"  
with my Algebra 1 students  
who spend part of a week researching topics  
& making a PowerPoint, book or short story.

-- 3/14/2008 email from Melissa S.,  
HS teacher in Nokomis, IL

# Appendices in my *JME* paper

(#7 is similar to *Holt Geometry* lab)

## Appendix 1: “American Pi” lyric

©1997-2015 Lawrence M. Lesser (reprinted with author’s permission)

*Prologue:* A long, long time ago I can still remember  
How that math sure used to make me smile.  
And I knew if I had my chance, I would ace geometry class  
And make my parents happy for a while.  
But some math books made me shiver--  
Dry procedures, all delivered:  
Nothing past the rational,  
And nothing transcendental.  
I can’t remember if I cried, reading 3.14159...  
But something touched me deep inside  
The day I learned of  $\pi$ ... so:

*CHORUS:* Find, find the value of  $\pi$ , starts 3.14159  
A good ol’ fraction you may hope to define,  
But the decimal never dies, never repeats or dies...

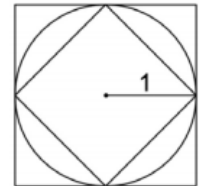
In the Bible we do see the circle ratio appears as 3,  
Or a little more....  
That genius Archimedes found with polygons, an upper bound  
Of  $22/7$  for sure!  
The Chinese got it really keen: three-five-five over one-thirteen!  
More joined the action with continued fractions.  
In the 1700s, my oh my, the English coined the symbol  $\pi$ ,  
Then Lambert proved that you and I can’t find rational  $\pi$ .  
He started singing.... (*Repeat Chorus*)

Late 1800s, Lindemann shared why a circle can’t be squared  
But some folks tried anyway--  
Like the Indiana doctor who said  $\pi$  was 4 or 3.2  
And thought his proof should be a law someday.  
The Indiana congressmen read his paper there and then  
A bill got through the House by a vote unanimous!  
But in the end, the statesmen cried, “It’s not for us to decide,”  
So the bill was left to die like the quest for rational  $\pi$ .  
Let’s try singing.... (*Repeat Chorus*)

That doctor’s pi-in-the-sky dreams may not look so extreme  
‘Cause we long believed  
Deductive systems could be complete and there was one true geometry.  
But now there’s more we see  
Now when it comes to  $\pi$ , we test the best machines to find  
Many trillion places that so far lack pattern’s traces.  
It’s great when we can truly see math as human history--  
That adds curiosity: easy as  $\pi$ !  
Let’s all try singing.... (*Repeat Chorus*)

## Appendix 2: class questions, in order of appearance in “American Pi”

1. What makes a number irrational and can you name one besides pi?
2. What makes a number transcendental? Can you name one besides pi?
3. What is the difference between a decimal and a fraction and what are the advantages of each?
4. What is an example of a fraction where the digits of its decimal representation terminate?
5. What is an example of a fraction where the digits of its decimal representation go into a repeating pattern?
6. How does 1 Kings 7:23 suggest a value of 3 for pi? (extension: see Lesser (2006) to learn how further information can yield a closer value for pi)
7. Recreate how Archimedes started his process by using appropriate tools to construct a circle of unit diameter (so the circumference will equal  $\pi$ ), and then find the perimeters of the inscribed and circumscribed hexagons. Those perimeters are lower and upper bounds for  $\pi$ .  
To explore a similar, but simpler, version of Archimedes’ process, explore this diagram and use the perimeters of the squares and the unit circle to verify that because the circumference of a unit circle is  $2\pi$ , then  $2\pi$  is bounded by  $4\sqrt{2}$  and 8, which places the value of  $\pi$  between 2.8 and 4.
8. Verify how many decimal places of accuracy the Chinese approximation 355/113 provides.
9. Look up what a “continued fraction” is. Find the simplified improper fraction represented by this truncated continued fraction:  $3 + 1/(7 + (1/(1/1)))$
10. Look up why the Greek letter  $\pi$  was an appropriate choice (by English mathematics teacher William Jones in 1706) of a symbol for the circle ratio.
11. With respect to the 1761 Johann Lambert proof that pi is irrational, consider the result that “if  $x$  is rational, then  $\tan(x)$  is irrational”. That is logically equivalent to the contrapositive statement “if  $\tan(x)$  is rational, then  $x$  is irrational”. Now let  $x = \pi/4$  and see if you can deduce that  $\pi$  is irrational.
12. Squaring the circle means constructing a square with the same areas as a given circle using a finite number of steps with the compass and straightedge. Given a circle with unit radius, what is the circle’s area? For a square to have that same area, what would the length of a side of the square have to be? Can the answer to that last question be the root of a polynomial with rational number coefficients?
13. Use Hallerberg (1977) to explain how 4 and 3.2 were obtained.
14. What is a deductive system and what did Kurt Gödel prove in 1931 about completeness?
15. For about 2000 years, Euclidean geometry was assumed to be the only “true” geometry. What is another geometry we now know to be just as valid



# covered by Calvin Coolidge (band of then-HS students in Cleveland)


small world connection: the bassist's dad's klezmer band played at my wedding the year I wrote the song!



## American Pi



stratton111

 **Subscribe** 3

8,760 views

 Add to  Share  More

 19  1

Uploaded on Mar 23, 2006  
Calvin Coolidge plays American Pi



# H. Lewellen's math club (Shaker Heights, OH) wrote song from first 50 digits of pi:

B	middle C	D	E	F	G	A	B	C	D
0	1	2	3	4	5	6	7	8	9

(see April 1987 *Mathematics Teacher*)

3 1 4 1 5 9 2 6 5 3 5 8 9 7 9 3 2

C F Em7 Dm G7 C Fmaj7 Em7 Dm7 G7

Math-e-mat-ics mu-sic holds a fan-ta-sy. Cir-cle 'round the an-swer;

3 8 4 6 2 6 4 3 3 8 3 2 7 9 5 0 2 8

C F G7 Fmaj7 Em7 Dm7 G7 G Dm7

soive the mys-ter-y. Mi-nus one is my co-sine; and my sine is ze-ro.

8 4 1 9 7 1 6 9 3 9 9 3 7 5 1

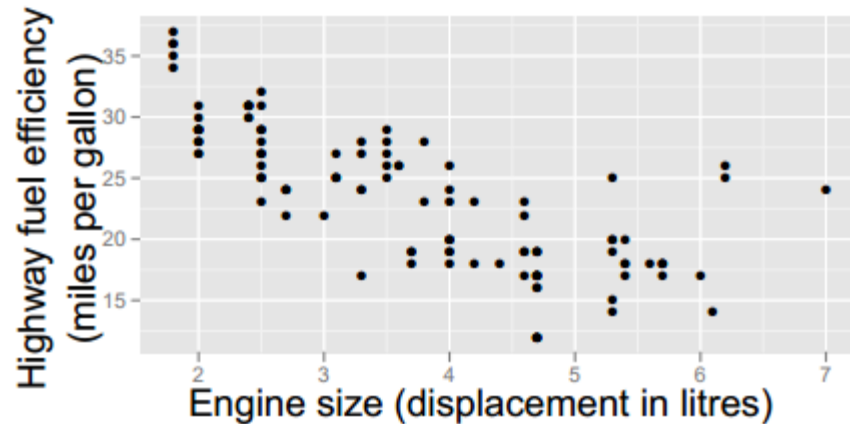
F G7 Fmaj7 Em7 G G7 C

My co-tan-gent's in-fi-nite. Can you guess my name? I am pi!

# You can get music not only from a number's digits, but also from data!

(Ethan Brown et al., 2012)

Fuel efficiency scatterplot: visualization



*Engine size* → *x*, *fuel efficiency* → *y*

Fuel efficiency scatterplot: sonification



*Engine size* → *time*, *fuel efficiency* → *pitch*

2016 interview in STEM songwriting blog captured a lot of my motivations/process:

<http://singaboutscience.org/wp/>

### STEM songster interview #19: Professor Lesser

Posted on February 18, 2016 by crowther



*November 2015: Larry Lesser prepares to play "The Gambler," one of his winning entries in the 2015 Museum of Mathematics (MoMath) song contest.*

One of the only people in this world who devotes as much time as me, or more, to writing educational STEM songs and thinking about how to use them effectively is Prof. Lawrence M. Lesser of The University of Texas at El Paso (UTEP). After years of occasionally corresponding with Larry via email, and reading his papers, I was excited to meet him in person for the first time when he recently visited Seattle — a visit which led to the following interview.

***Sing About Science & Math:*** *This interview, conducted via email, is an extension of a conversation we began in Seattle. Larry,*



# feature in *The Mini Page* (for Music in the Schools Month)

March 12-18, 2016



## The Mini Page

Issue 11, 2016

Founded by Betty Debnam

**Next Week:**  
Giant panda update



**Music = Math!**

### Mini Quote:

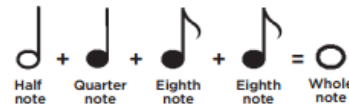
"Music is the pleasure that the human soul experiences from counting without being aware that it is counting."

—German mathematician  
Gottfried Wilhelm von Leibniz

### Graphing music

We can see that sheet music notation is really just a graph. Written music graphs two variables: the length of time (duration) of each note, and the pitch (frequency) of that note.

#### Home on the Range



March is Music in Our Schools Month, but we don't have to limit music to just one month or to music class. Most concepts in music make *noteworthy* connections to math!

### Rhythm and arithmetic

You have sung many songs without realizing you are counting beats with a certain pattern. For example, clap along while you sing "Rock-a-Bye Baby." Notice how the syllables of the lullaby naturally suggest counts of 1-2-3 as you sing it, even though some syllables get more than one beat.

### Fractions in music

We say "Rock-a-Bye Baby" is in "three-quarter time" because each measure, marked off by vertical lines in the sheet music, contains the equivalent of three quarter notes. (The time signature visually resembles the fraction  $\frac{3}{4}$ .) Other songs have different rhythmic patterns. For example, a popular song in "four-four time" (where each measure gets the equivalent of four quarter notes) is "Twinkle, Twinkle Little Star."



Notes can be whole notes, or half, quarter, eighth, 16th, 32nd or 64th notes. In  $\frac{4}{4}$  time, a whole note gets four beats.

Using fraction math, you can see that  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = 1$ , just as four quarter notes would also add up to 1. Can you think of other fractions that would add up to 1?

### Harmony and fractions

Borrow a guitar from a teacher or friend. The length of a guitar string affects the pitch of the note the string makes.

Choose one of the strings and pluck it, then play that same string while holding it down against the fretboard so that only half of the string's length is free to vibrate. You notice not only the mathematical relationship that shortened strings have a higher pitch, but the two notes also sound the same, yet different.

Harmony involves fractions. The string lengths are in a 2-to-1 ratio, and the shorter length vibrates twice as much as the longer length. This produces the sound of an octave.

What other words does octave remind you of? Octagon? Octopus? If you write out the major scale (do, re, mi, fa, so, la, ti, do), the "low do" and "high do" span eight notes. (Play the white piano keys from C to the next highest C, for example.)

The low C and the high C are the same note, but different octaves.



### Math songs

Not only is there math in music, but you can also bring music to math by singing (or writing!) songs about math.

The National Museum of Mathematics has held contests for math teachers and students to write math songs. Try taking a song you know and changing the words or adding onto it to illustrate whatever math concepts you are now learning in school.

### Resources



#### On the Web:

- [bit.ly/InE0pbm](http://bit.ly/InE0pbm)
- [bit.ly/1RYFEUO](http://bit.ly/1RYFEUO)
- [momath.org](http://momath.org)

#### At the library:

- "Music Math: Exploring Different Interpretations of Fractions" by Kathleen Collins

# Outline

- Background
- *Addressing Hesitations*
- Finding Songs
- Using Songs
- Writing Songs
- Interactiveness
- Q&A

# What might be hesitations, given all these potential benefits?

Multidisciplinary connections,

Memory aid,

Multiple intelligences,

Motivation,

Making community,

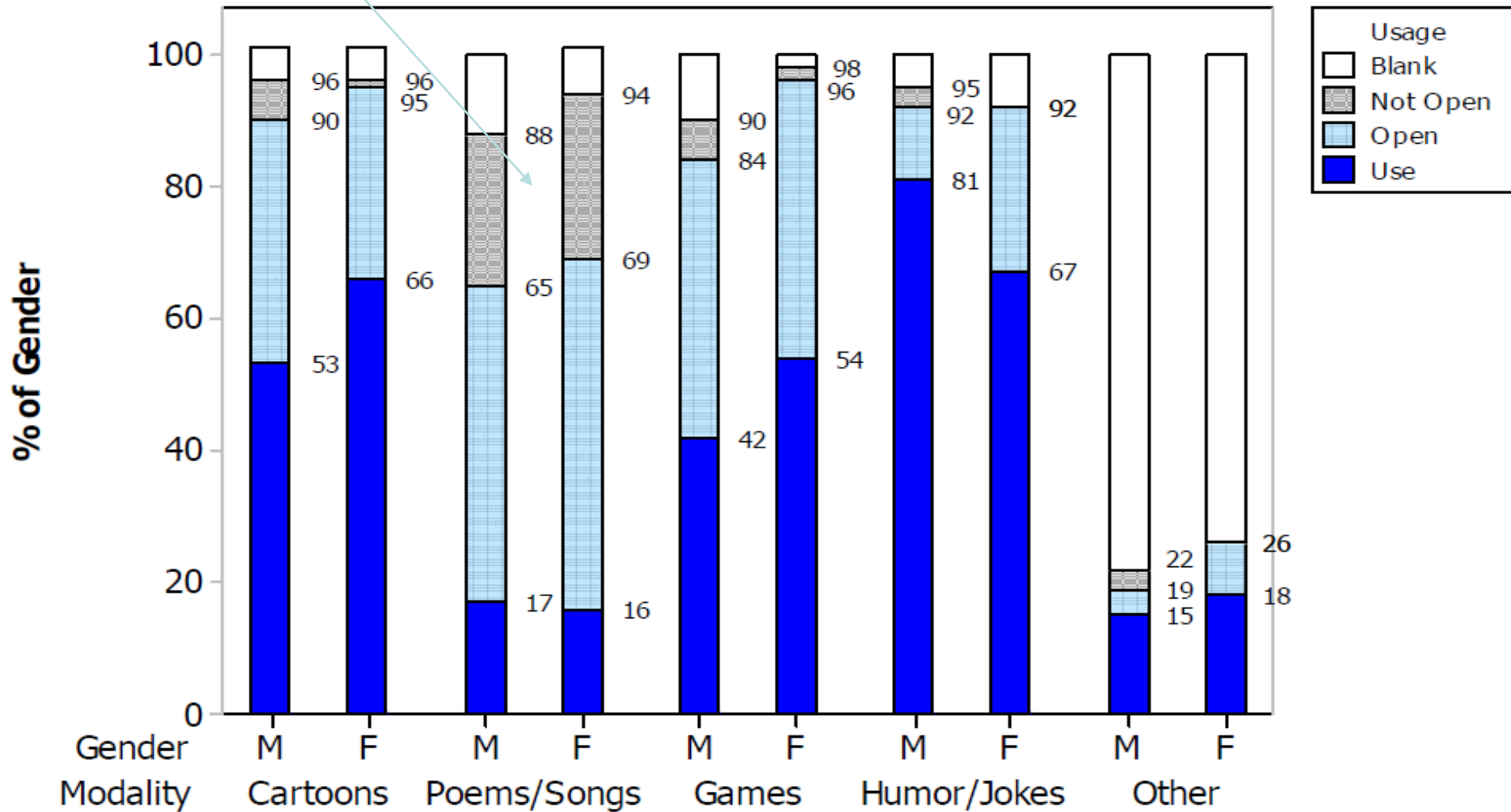
Math anxiety reduction,

Mashing stereotypes (content, class, teacher),

Modelling stretching, etc.

# (N=249) statistics instructors (Lesser et al., 2013)


## less open to using song than other fun



**Figure 1.** Instructor Openness to Different Modalities of Fun

# addressing hesitations

found in Lesser et al. (2013)

Hesitations	Solutions
Can't quickly find good examples	CAUSEweb.org, etc.
No skills/talent	Press "PLAY"; tap student talent
Uses too much time	Use as students enter or papers handed back. Assign as HW. Streamline length. Have students access online outside class.
Incompatibility with students' cultures	Know your audience (week 1 survey, etc.)
Class size	Song works just as well in large classes
Need to be seen as serious by students	Make explicit connections to content (or assessment); make a mini-lesson plan
Need to be perceived as serious by colleagues/supervisor; weak evidence of helping learning	cite research studies, calls for  engaging/active learning, etc.

# a classroom study (VanVoorhis, 2002)

## 2 equal-GPA sections of stats

Section **R** read 3 definitions aloud;

Section **S** sang 3 stat jingles (for same concepts)

S did better than R on relevant test items

**$p < .05$**

S had high correlation between test score and self-rated jingle knowledge

**$p < .05$**

# $p$ -value DEFINITION vs. SONG:

A  $p$ -value is the probability you obtain at least this strong of a difference if the null hypothesis of no effect were really true.

It is key to know what  $p$ -value means – It's the chance (with the null) you obtain data that's at least that extreme!

lyric © 2005 L. Lesser



# in July 2016 *J. of Statistics Education*

JOURNAL OF STATISTICS EDUCATION  
2016, VOL. 24, NO. 2, 54–62  
<http://dx.doi.org/10.1080/10691898.2016.1190190>



OPEN ACCESS

## Assessing Fun Items' Effectiveness in Increasing Learning of College Introductory Statistics Students: Results of a Randomized Experiment

Lawrence M. Lesser<sup>a</sup>, Dennis K. Pearl<sup>b</sup>, and John J. Weber, III<sup>c</sup>

<sup>a</sup>Department of Mathematical Sciences, The University of Texas at El Paso, El Paso, Texas, USA; <sup>b</sup>Department of Statistics, The Pennsylvania State University, University Park, Pennsylvania, USA; <sup>c</sup>Department of Mathematics, Computer Sciences, and Engineering, Perimeter College at Georgia State University, Clarkston, Georgia, USA

### ABSTRACT

There has been a recent emergence of scholarship on the use of fun in the college statistics classroom, with at least 20 modalities identified. While there have been randomized experiments that suggest that fun can enhance student achievement or attitudes in statistics, these studies have generally been limited to one particular fun modality or have not been limited to the discipline of statistics. To address the efficacy of fun items in teaching statistics, a student-randomized experiment was designed to assess how specific items of fun may cause changes in statistical anxiety and learning statistics content. This experiment was conducted at two institutions of higher education with different and diverse student populations. Findings include a significant increase in correct responses to questions among students who were assigned online content with a song insert compared with those assigned content alone.

### KEY WORDS

CAUSEweb fun collection;  
Humor; Song; Statistics  
education research

# student-randomized experiment (fall 2013)

(53 from comm. college; 194 from medium-sized university)

- Half the students randomized to always have “fun inserts (song, cartoon, etc.)” in content mini-readings accessed via LMS
- All students take exams with 12-14 embedded MC items related to the readings, such as:

If data points fit tightly around the line  $Y = 2 - 3X$ , we can conclude there is \_\_\_\_\_ correlation.

- A. positive
- B. negative
- C. zero
- D. more information needed

- All students asked to take pre/post surveys: attitude & anxiety

# % Correct with and without Song Inserts




Topic	Without song	With Song	Difference
Margin of error: down with $n$ down by $\sqrt{n}$	57.3%	61.3%	4.0%
	9.1%	10.0%	0.9%
Standard score	62.5%	75.0%	12.5%
<b>Correlation &amp; slope</b>	<b>60.2%</b>	<b>73.8%</b>	<b>13.6%</b>
Equiprobability bias	40.9%	50.0%	9.1%
Multiplicity	36.1%	37.0%	0.9% (medium university)
$p$ -value	44.4%	50.0%	5.6% (2-yr. college)
<b>OVERALL</b>	42.3%	50.0%	7.7%
$p$ -value $\approx 0.04$			

# Outline

- Background
- Addressing Hesitations
- *Finding Songs*
- Using Songs
- Writing Songs
- Interactiveness
- Q&A

# besides YouTube.....?

The image shows a screenshot of a YouTube playlist page. At the top, there is a search bar with the text 'Search' and a magnifying glass icon. Below the search bar is a large video player area. On the left side of this area, there is a vertical list of names: Mike Andrejkovics, Sun, Ro, Sun, Stev, Sun, Per, Ste, Sun, Pete, Sun, Tim, Sun. The main video player shows a man in a white shirt and tie speaking into a microphone. To the right of the video player, the text reads 'MoMath - The Open Set' and 'MikeDobbs76'. Below this, there is a 'PLAY ALL' button. Underneath the playlist title, it says '14 videos • 1,922 views • Last updated on Dec 6, 2015'. The playlist contains three items:

- 1  **MoMath Math in NYC Trailer**  
MikeDobbs76  
0:36
- 2  **LIVE Performance at MoMath in NYC!**  
MikeDobbs76  
4:02
- 3  **Order it's done - LIVE at MoMath NYC**  
MikeDobbs76  
4:27

# my [math/stat](#) song resource page

just Google: “Larry Lesser” or “Mathemusician”

[www.math.utep.edu/Faculty/lesser/Mathemusician.html](http://www.math.utep.edu/Faculty/lesser/Mathemusician.html)

<a href="#">Resources &amp; Bibliography</a>	<a href="#">SONGS!</a>	<a href="#">Cool Math &amp; Music Quotes</a>
<a href="#">More on Lesser</a>	<a href="#">Presentations &amp; Press</a>	<a href="#">Contact Info</a>



## *The Mathemusician*

“Find, find the value of pi -- starts 3 point 1 4 1 5 9.

A good ol’ fraction you may hope to define, but the decimal never dies ....”

-- from Larry Lesser’s award-winning “[American Pi](#)”



**ABOUT THE MATHEMUSICIAN:** To support his mission to motivate learning mathematics/statistics, Professor Lesser integrates two great loves – math and music! An award-winning songwriter in general songwriting contests, Lesser has also won awards for educational songs and song videos in recent national contests ([ASA](#), [National Museum of Mathematics](#), [CAUSE](#), [QL-SIGMAA](#)) and a [video of songs was shown at Bridges 2014](#). His songs have yielded statistically significant learning gains in an [NSF-funded randomized experiment](#), helped

# statistics songs

causeweb.org/cause/resources/fun/

- site launched in 2005 (part of NSF's National Science Digital Library)
- searchable fun collection now has 600+ items (including **100+ songs** almost all with MP3s), 9 modalities, bibliography, & lesson plan guidance

The screenshot shows the CAUSEweb website interface. At the top, there is a navigation bar with the CAUSE logo and menu items: HOME, CONFERENCES, PROF DEV, RESEARCH, RESOURCES, and AB. A search bar is located below the navigation bar. The main content area is titled "CAUSEweb Fun Collection" and includes a brief introduction to the collection. Below the introduction, there are search filters for Keyword, Type, Topic, and Items per page. The "Type" dropdown menu is open, showing options: Any, Cartoon, Gallery, Joke, Magic, Poem, Puzzle, Quote, Song (highlighted), and Video. The "Topic" dropdown is set to "Estimation". The "Items per page" dropdown is set to "15". An "APPLY" button is visible. Below the filters, a song titled "Call It Maybe" is displayed, with lyrics and a note about its origin.

Secure | https://www.causeweb.org/cause/resources/fun/all?keys=&type\_1=song&field\_tags\_tid=10&items\_per\_page=15

Register

CAUSE

HOME CONFERENCES PROF DEV RESEARCH RESOURCES AB

Search for... CAUSEweb

### CAUSEweb Fun Collection

In this section we provide fun items related to statistics, including cartoons, a gallery of statistical art, jokes, magic, poems, word puzzles, quotes, songs, and videos. Click [here](#) for an introduction to the literature and some usage guidance on teaching statistics using fun items. Consider participating in the A-μ-Sing contest to contribute to this collection. see [recent winners](#) and [contest rules](#) for more information.

Displaying 1 - 8 of 8

Keyword Type Topic Items per page

Estimation 15 APPLY

Call It Maybe

Lyrics ©2013 by Lawrence Mark Lesser

be sung to the tune of 'Call Me Maybe' (Carly Rae Jepsen, Tavish Crowe, and Josh Ramsay)

I went to find people's heights. I measured each person thrice:  
The numbers didn't match nice - and I said 'Oy vey!'  
When I was asked for the mean, I knew I had to come clean:



# science/math songs

<http://singaboutscience.org/>

SEARCH OUR DATABASE OF 7000+ SONGS

Keyword/Phrase:

Keyword/Phrase:

Performer/Songwriter:

Song Title:

Song Template:

(i.e., the original song upon which a parody is based)

Album Title:

Website URL:

(e.g., youtube.com [omit http://www.]

## Additional (optional) search restrictions:

- Only include songs intended for students between ages  and  ages" or adults will not be included if the range is narrowed to less than 0-9
- Only include songs known to be parodies.
- Only include songs with online lyrics.
- Only include songs with free full-length online recordings.
- Only include songs with free full-length online videos.
- Don't include songs that may contain adult language.

# Outline

- Background
- Addressing Hesitations
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- *Using Songs*
- Writing Songs
- Interactiveness
- Q&A

# Roles of Song (Lesser, 2014)

- recall
- introduce concepts/terms
- reinforce thinking process
- connect to history/real-world
- humanize

# Things to consider

- Alignment with content learning objectives and standards (CCSSM? PSSM? GAISE?)
- Setup or pre-assessment
- Delivery: in class (a capella, live instrument, karaoke, or MP3?) or offline?
- Student level of participation
- Follow-up or post-assessment/exploration (e.g., the “American Pi” questions)

# tips from Brame (2016)

TABLE 1. Practices to maximize student learning from educational videos

Element to consider	Recommendation	Rationale	Examples
Cognitive load	Use signaling to highlight important information.	Can reduce extraneous load. Can enhance germane load.	Key words on screen highlighting important elements Changes in color or contrast to emphasize organization of information Changes in color or contrast to emphasize relationships within information Brief out-of-video text explaining purpose and context for video (e.g., learning objective for video)
	Use segmenting to chunk information.	Manages intrinsic load. Can enhance germane load.	Short videos (6 minutes or less) Chapters or click-forward questions within videos
	Use weeding to eliminate extraneous information.	Reduces extraneous load.	Eliminating music Eliminating complex backgrounds
	Match modality by using auditory and visual channels to convey complementary information.	Can enhance germane load.	Khan Academy-style tutorial videos that illustrate and explain phenomena Narrated animations
Student engagement	Keep each video brief.	Increases percentage of each video that students watch; may increase total watch time. May decrease mind wandering.	Multiple videos for a lesson, each ≤ 6 minutes
	Use conversational language.	Creates a sense of social partnership between student and instructor, prompting the student to try harder to make sense of the lesson.	Placing the student in the lesson by use of “your” rather than “the” during explanations Use of “I” to indicate the narrator’s perspective
	Speak relatively quickly and with enthusiasm.	Increases percentage of each video that students watch. May increase sense of social partnership between student and instructor.	Speaking rates in the 185–254 words per minute range Expressions of instructor excitement, such as “I love the next part; the way the feed-forward mechanism works is so elegant,” or “Consider how the cell solves this tricky problem of needing to regulate three genes in sequence; it’s really cool.”
	Create and/or package videos to emphasize relevance to the course in which they are used.	Increases percentage of each video that students watch. May increase germane cognitive load by helping students recognize connections.	Videos created for the class in which they are going to be used, with instructor narration explaining links to preceding material Explanatory text to situate video in course
Active learning	Consider these strategies for promoting active learning: Packaging video with interactive questions.	May increase germane cognitive load, improve memory via the testing effect, and improve student self-assessment.	Integrate questions into videos with HapYak or Zaption, as described by Obodo and Baskauf (2015) Follow short videos with interactive questions within an LMS, as done by Keithly and colleagues (2015), or within Google Forms, as done by Caudel and colleagues (2015)
	Use interactive features that give students control.	Increases student ownership and may increase germane cognitive load.	Create “chapters” within a video using HapYak or YouTube Annotate
	Use guiding questions.	May increase germane cognitive load, reduce extraneous cognitive load, and improve student self-assessment.	Senchina (2011) provides guiding questions for videos designed to introduce physiology students to professional ethics related to experimenter–subject interactions, such as the following: “Observe the subject’s behavior and responsiveness during the dehydration period. What changes as the subject becomes dehydrated? What problems does he have? Observe the experimenters’ behavior and responsiveness as dehydration progresses. What do they do differently? Why?”
	Make video part of a larger homework assignment.	May increase student motivation, germane cognitive load, and student self-assessment.	Package videos with a series of questions or problems that ask students to apply the concepts from the videos. iBiology Education videos (e.g., <i>What Can You Learn with a Light Microscope?</i> ) provide one example (iBiology, 2016)

# Critical Reflection: teaching/performing parallels

- **Awareness** – your style/strengths, your audience's backgrounds
- **Preparation** – so you have enough attention to respond to nonverbal cues; what needs practiced precision and what is better freshly improvised?
- **Density** – when 'less is more'; finding spaces for audience interaction/contribution
- **Progression** – meaningful sequence (beyond isolated songs/slides) that has culmination

we want: significant “fun value”  
that does not overshadow educational content

I agree with Resnick (1987) that:

we should avoid implying that education is a  
“bitter medicine that needs the sugar-coating  
of entertainment to become palatable”

or that education and entertainment are  
“services that someone else provides for you”  
rather than “things that you do”



# Diversity connections

- Song affected by specific diversities of instructor and campus population; e.g., it may not work for teacher to use a rap persona, or parody a hymn or “Yellow Rose of Texas”
- Avoid harmful stereotypes (see my April 2014 *Mathematics Teacher* op-ed)
- Ethical/respectful fun builds community!

coming....Sept. 27-28!  
(STEM-focused, almost free,  
pedagogy/research/practitioner angles)

info: [crowther@uw.edu](mailto:crowther@uw.edu)



# VOICES

**2017** Virtual Ongoing Interdisciplinary  
Conferences on Educating with Song

# Outline

- Background
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- Using Songs
- *Writing Songs*
- Interactiveness
- Q&A

# Who wrote this: a mathematician or songwriter?

“the more you are familiar with **problem solving**,  
you recognize variations  
on problems that you've solved before....

The typical problems,

everyone learns to solve in the early years...

But with the atypical problems,

experience and technique become invaluable.”

# Who wrote this: a mathematician or songwriter?

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you recognize variations  
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The typical problems,

everyone learns to solve in the early years...

But with the atypical problems,

experience and technique become invaluable.”

**Paul Simon!**

(interviewed in Sept./Oct. 2006 *Performing Songwriter*)

# parallels: doing math & songwriting

- “Flashes of inspiration” may come anytime, but are usually preceded by much reflection, false starts, and prior effort of developing a toolkit and repertoire
- Essence comes fast, details take longer
- Versions can have same “content”  
but differ greatly aesthetically
- Some are steps toward something “bigger”
- Useful to try varying structures or starting points
- Peer feedback/collaboration helpful!
- Cultivate “whole-brain”, not just left or right-brain

# song (title) ideas

- Songs with numbers or math terms (modulo a pun): several songs have used “50 Ways to...”
- Steven Galbraith’s “List of Unintentionally Mathematical Songs”
- *The Green Book*
- SongSet.net
- Common misconceptions (my “1 in 2”, “Losing Cause”, etc.)



# Algebra text exercise

(Wells & Schmitt 1996)

## 7-2 A Graphical View of Polynomial Functions

321

71. (Review)  $f(x) = -2x^6 + 24x^4 - 96x^2$

72. (Review)  $f(x) = x^4 - 14x^3 + 24x^2$

73. (Review)  $f(x) = x^6 - 12x^3 + 100$

74. (Review)  $f(x) = x^7 - 1000x$

75. "From a distance, the world looks blue and green,  
And the snow-capped mountains white."

*From a Distance*  
Julie Gold

(Writing to Learn) Graph each of the functions  $f(x) = x^7$  and  $g(x) = x^7 - 3x^6 + x^5$  in the window  $[-100, 100]$  by  $[-10^{14}, 10^{14}]$ . These graphs provide visual support for the Highest Degree Theorem. Explain why. (What does the line from the song have to do with the exercise? You may want to use it in your explanation.)

some of my math song titles	topic
Mean	Measure of center
The Gambler	lottery literacy
From a Distance	graphing
American Pie <del>Pie</del> Pi	History of the number pi
Hotel California <del>Called</del> Infinity	exploring infinity via Hilbert's Hotel
Call Me <del>It</del> Maybe	variation
<del>Home on the</del> Domain and Range	domain and range
Hit Me with Your Best <del>Shot</del> Plot	graphing
<del>Imagine</del> Imaginary	imaginary numbers
<del>Livin' La Vida Loca</del> Findin' Extrema Local	finding local extrema

# disciplinary songwriting challenges

- Technical precision versus accessibility/singability
- Is jargon singable and rhyme-able?
- Are math terms in important (repeating or rhyming) positions?
- Balance concrete and abstract
- Keep tone inviting; don't just address insiders
- Align to specific learning objective (e.g., not just a “math is cool” song, or a song that includes math words but not in a way that teaches)

# Difficulty continuum in song forms

- Parody (change some words in existing song)
- Rap (rhythm and rhyme, but no melody)
- 12-bar blues
- 1-verse jingle
- AAA
- AABA
- ABAB (Verse-Chorus)

# Lyrics need more than rhyme

(Lesser, 2014)

- **Melody** conveys meaning
- Letter combinations must be **singable**, and have grammatical phrasing aligning with musical pauses
- Must have **conversational** (e.g., subject-verb-object) word order and avoid ambiguous homonyms and long clauses to communicate by ear in real time on first listening
- **Place key words** where there are end rhymes, long notes, downbeats, or repetition

# free (or nearly free) song apps

- Rhymezone.com, Rhymmer.com, b-rhymes
- GarageBand
- SoundCloud
- Evernote, FourTrack, Songwriter's Pad, Suggester, Simple Songwriter, Nanostudio, Song Box Lyric Writer, Songboard, Hum, Band Creator, SongSet, etc.

# student-made examples

- **Class project/enrichment:** rubric can assess creativity, accuracy, how well lyrics go beyond just mentioning a term, etc.; students can put song on YouTube, SoundCloud, etc.
- **Contest:** National (e.g., MoMath, CAUSE) or classroom
- **Benefits** may be similar to what Young et al. (2013, p. 15) found with poetry:





# Young et al. (2013, p.15)

“provides a voice to some students who are shy about speaking in class;

it provides a place for humor, playfulness, irreverence, and the expression of shared emotions;

it recasts thinking about course material into new patterns;

it enhances student-teacher and student-student communication; and it fosters an interactive classroom environment”

# Outline

- Background
- Addressing Hesitations
- Finding Songs
- Using Songs
- Writing Songs
- *Interactiveness*
- Q&A

# a continuum of song “interactiveness”

- Listen to recording
- Listen to performer
- Provide accompanying rhythm
- Sing along (with lyric displayed)
- Complete each rhyme with only 1 right answer
- Provide inputs to help create the song
- Write your own song (with scaffolding offered)

2015 - present    NSF grant (DUE 1544237)

# Project

**Student-Made Interactive Learning with Educational Songs** (for introductory statistics)

GOAL: to write (and assess) interactive songs  
of high aesthetic and pedagogical quality  
to maximize learning and engagement

# why statistics is a great vehicle for our NSF songs grant

- **Statistics** is offered in several departments and taken by students in many disciplines
- **Statistics anxiety** occurs across disciplines (e.g., in the humanities, social sciences, and in classes for pre-service elementary school teachers), and anxiety-reduction is one potential benefit of the use of fun items

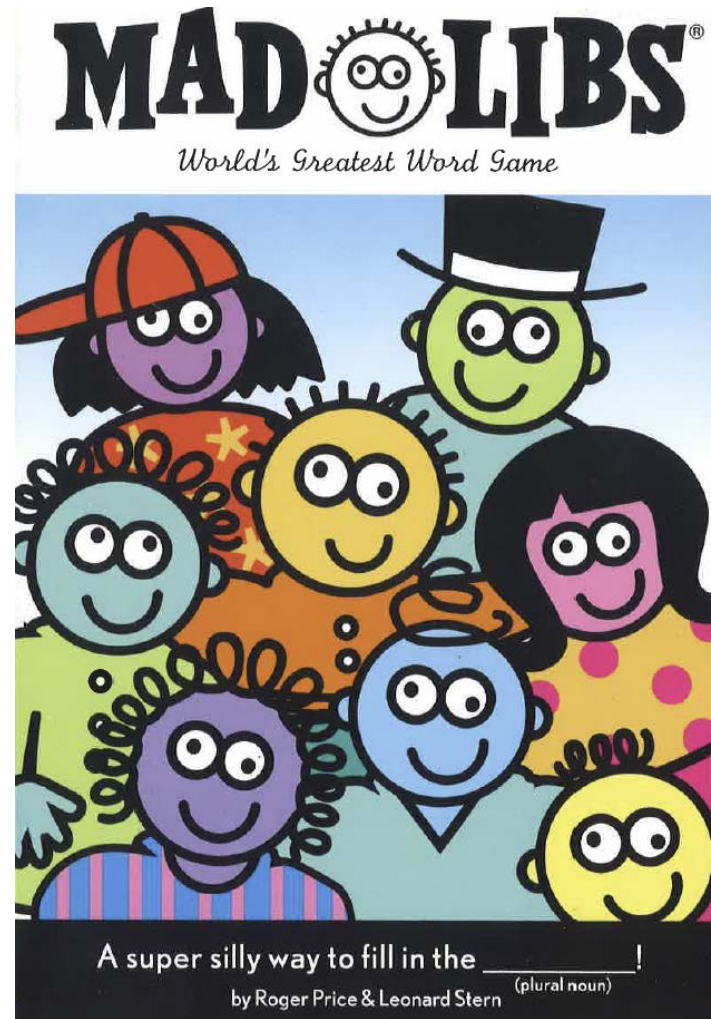
*Quick! I need...*

An exclamation

An adverb

A verb ending in 'ing'

A plural noun



the Mad Lib result:

“\_\_\_\_\_!” she said \_\_\_\_\_.

*exclamation*

*adverb*

This talk on song is as noteworthy as

\_\_\_\_\_!

*verb ending in 'ing'*

*plural noun*

# Writing songs ready for inputs!

- “Inputs” can be neither trivial nor too open-ended, whether they make a conceptual connection or provide an example
- Inputs generally must avoid end-rhyme  
*“How do **shoe length** values go  
When **height** is high or when it’s low?”*
- Inputs need to be robust or scaffolded for part of speech, verb tense, (rough) number of syllables, etc. *“....When **elbow length** is high or low?”*
- Keep short/interesting to keep engagement
- Can’t just take an existing song and remove a few words



# topics of 22 songs (aligned with literature,

**Guidelines for Assessment and Instruction in Statistics Education, Goals and Outcomes Associated with Learning Statistics instrument)**

- Levels of measurement
  - Mean vs. median
  - Convenience vs. random sampling
  - Patterns of correlation
  - Correlation vs. causation
  - Statistic vs. parameter
  - Estimator bias
  - Margin of error in poll
  - Effects on width of CI
  - framework of testing  $H_0$
  - Reporting test conclusion
  - Concepts of  $X^2$  test
  - Effect of  $n$  on significance
  - Concepts of regression model
  - Concepts of ANOVA test
  - Variances (not SDs) add
  - Bayesian reasoning (most tests for rare traits yield false positives)
  - Central Limit Theorem
  - Simpson's Paradox
  - Ethics in statistics
- most are 2 min. long*

# songs written fall 2015 by collaborative of statistics and music professionals...

- **Greg Crowther** (Seattle, WA) – university biology lecturer, researches in science ed (STEM songs) and science, curates [singaboutscience.org](http://singaboutscience.org) database of 7000+ songs
- **Monty Harper** (Stillwater, OK) – award-winning full-time touring performer/writer of educational science songs; MS in mathematics
- **Tom Toce** (NYC) – theater/cabaret songwriter with album cuts and ASCAP awards; senior actuarial adviser at Ernst & Young; Fellow of Casualty Actuarial Society
- **Amy Adler Williams** (Austin, TX) – songwriter, cantorial soloist, music teacher, CPA, Certified Fraud Examiner

# ...and internal collaborators

(both at UTEP since 2004)

- **Larry Lesser** – statistics education researcher; award-winning songwriter; published 70+ math/stat lyrics and several papers on using music in statistics/math courses
- **Dominic Dousa** – classical music theory and composition faculty; (piano) accompanist and chamber musician; MS and college teaching experience in statistics



and then, UTEP's Steven Haddad

coordinates talented music majors of UTEP  
in its state-of-the-art Commercial Recording studio!



# pre-song prompts for “Simpson’s Paradox” song

Please fill in the blanks below with words that are **as short as possible**. Refer to these examples if you need help. Show examples: [Example 1](#) [Example 2](#) [Example 3](#) .

1. Give the name of a group that people could leave or join, comprised of two mutually exclusive types of people or individuals. [Hint](#)

2. Give a general label for any individual in your group (plural noun). [Hint](#)

3. Give a label (plural noun) for one type of individual in your group, ideally a type likely to score the higher average measure of your variable. [Hint](#)

4. Give a label (plural noun) for a second type of individual in your group, ideally a type

# pre-song prompts for “Super Bowl Poll” song

## Super Bowl Poll

1. Pick your favorite NFL team; if you don't have a favorite, just pick a team you think might be good:  .
2. The margin of error for a **sample proportion** for a survey of 1000 people would be about  %.
3. If 17% is the **sample percentage**, then the **margin of error** you entered in the above item gives an interval estimate as low as  % and as high as  %.
4. If you multiplied the **sample size** by a factor of nine, that would  the **margin of error** by a factor of  .

increase

decrease

Continue 

# Super Boll Poll

Will the Texans win next season's Super Bowl?

We asked about 1,000 fans in a scientific poll.

The margin of error was 3 %

That's roughly the reciprocal of the square root of  $n$ .

17% answered 'YES' in the poll,

But what could it be for the population as a whole?

At the 95% level of confidence

The interval goes from 14 to 20 %.

If we multiply the sample size by a factor of 9,

The new margin of error that we could find

Would be a third as large as what we had before.

Thanks to the formula, you know the score.

Go Texans !

---

The margin of error was 3 %

STUDIO VERSION



00:18

# Timeline – stay tuned!

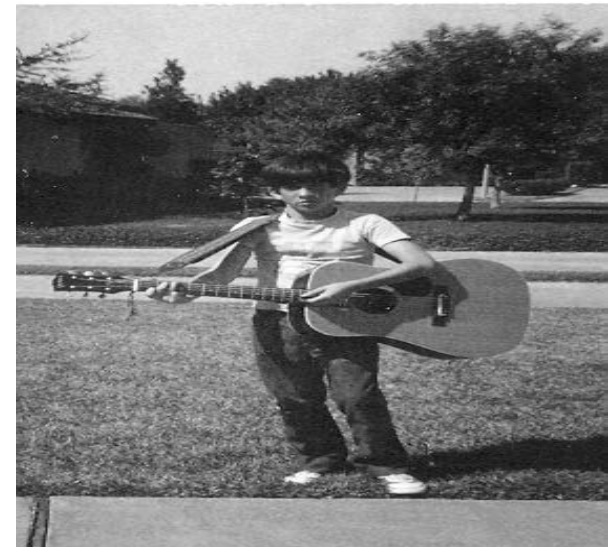
- Write lyrics, music, and template prompts; get peer feedback
- Pilot: do students understand prompts for inputs?
- Record songs in UTEP's Commercial Recording studio, then interface with singing voice synthesizer software
- Train automated grading system with several hundred students
- Experiment: students randomized to (1) readings only, (2) readings with complete songs, or (3) readings with interactive songs that they complete
- Post definitive completed versions of songs on CAUSEweb



# some *thank-you's*

Song An, Art Benjamin, Dane Camp, CAUSE, Greg Crowther, GCTM, Sarah Glaz, Kerry Go, Gizem Karaali, Ed Loewenstein, Leo Mazow, NSF, Bruce Palka, Dennis Pearl, Rey Reyes, Allan Rossman, Bill Robertson, Justin Slauson, Norm Sperling, Daniel Tillman, John Weber, Alvin White.

Mom & Dad for the first guitar & lessons!



# Annalisa Crannell

MAA Haimo Awardee for Distinguished Teaching (1/8/2008)

“Paradoxically, if our own intellectual lives are ‘all math, all the time,’ we face a lot of challenges when we try to woo others into our discipline.”

“our mania for ‘balancing’ parts of our lives (teaching & research; work & family) compartmentalizes them in harmful ways ...it’s possible to **integrate—rather than merely balance**” (1/8/2008)



*thanks for attending*

“Changing the Rhythm of Math Class: Using Educational Songs to Cultivate Learning and Community”

*WE WELCOME YOUR QUESTIONS & FEEDBACK*



**Larry Lesser**

The University of Texas at El Paso

[Lesser@utep.edu](mailto:Lesser@utep.edu)

<http://www.math.utep.edu/Faculty/lesser/Fun.html>

Join us at VOICES Sept. 27-28!

# Choose a song....

- Example of math rap (96)
- “American Pi” (104)
- “Mean” (110)
- “The Gambler” (113)
- “Hotel Called Infinity” (117)
- “World’s Longest Math Song” (123)

if not yet ready to try melody....

Give **rap** a ~~chants~~ chance --

Rap still has rhythm 'n' rhyme

short examples:

Defining trig functions: SOH-CAH-TO-A

4 rigid motions: turn, slide, flip & glide

Rhombus: Equilateral quadrilateral

Euler's formula:  $F + V = 2 + E$

# The Algebra Teacher Writes InVerse

(Lesser, 2017)

Triple  $x$ , then add 5, then divide by 2.

Label that answer  $y$ , now let's flip the view.

Double  $y$ , take off 5, third makes déjà vu!

$x$   $\xrightarrow{\times 3}$   $\xrightarrow{+5}$   $\xrightarrow{\div 2}$   $y$   $(3x + 5)/2 = y$

$y$   $\xleftarrow{\times 2}$   $\xleftarrow{-5}$   $\xleftarrow{/3}$   $x$   $(2y - 5)/3 = x$

Now .... *try this (algo)rhythm:*

Stomp-stomp-clap-(rest)

1

2

3

4

# For graphing $y = mx + b$ :

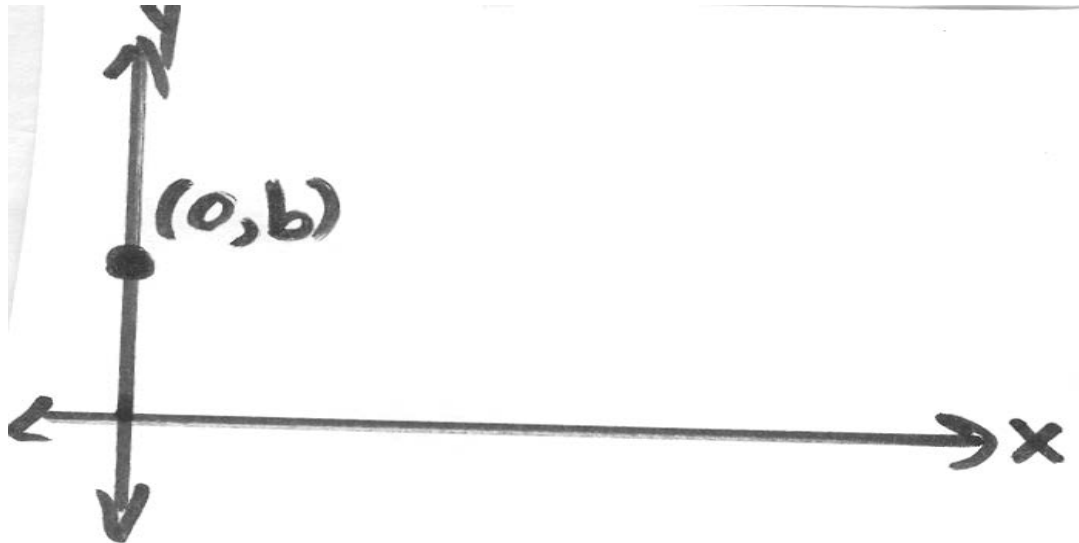
L. Lesser adaptation of 1998 version by former IL teacher-of-the-year & NCTM Board member John A. Carter

Buddy you're tryin' to draw the line

For the  $m$  and  $b$  we're assigned:

Ain't hard you see, put your pencil on the  $b$ ,

One more point will guarantee that....





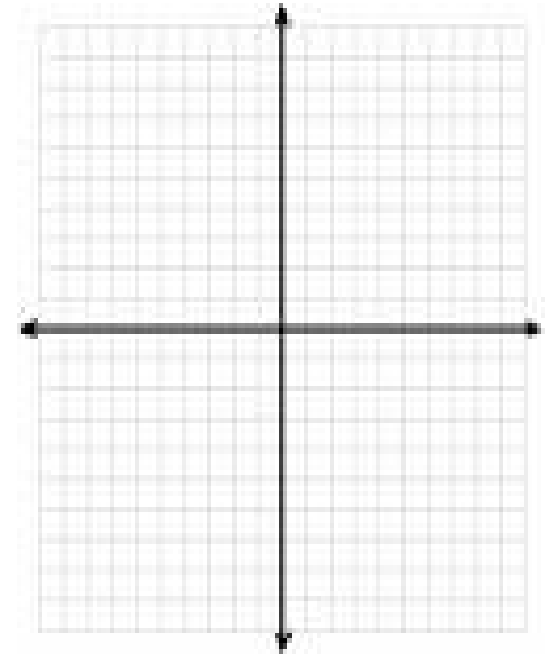
*sing-along:*

We will, we will

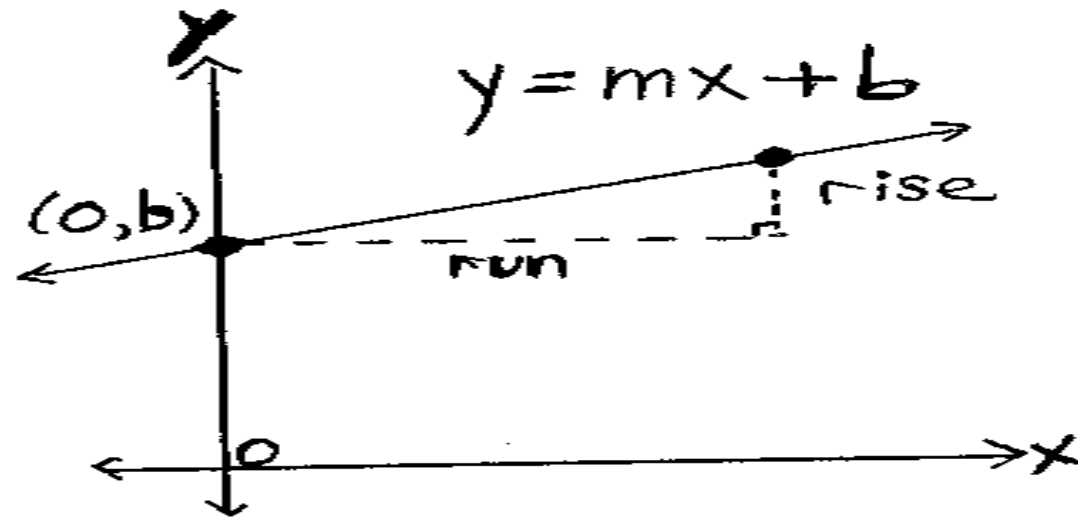
Graph you!

We will, we will

Graph you!



So now you know the  $y$ -intercept;  
Here's how we find the rest of it:  
Up with rise, across for run,  
Plot another point & when they're  
strung, sing:



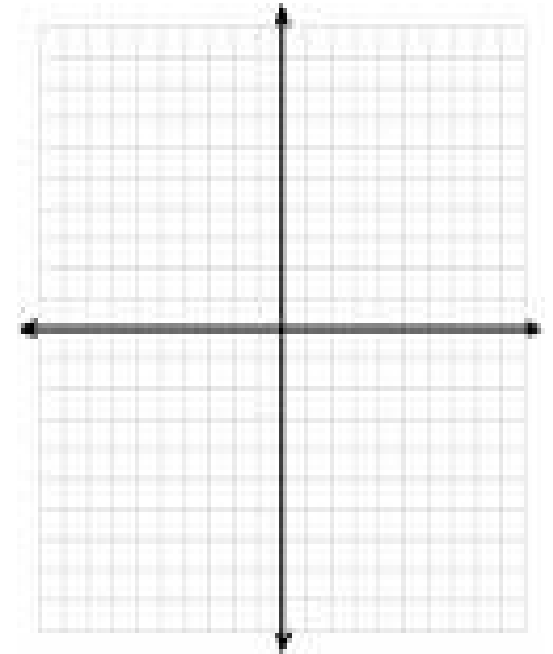
*sing-along:*

We will, we will

Graph you!

We will, we will

Graph you!



# Sound way to learn maths

A teacher puts complex maths theories to music and SAMANTHA AMJADALI finds it adds up

Larry Lesser has discovered the secret to fostering a love for mathematics in his students — Kenny Rogers' catchy ballad *The Gambler*.

Dr Lesser, a mathematician with a penchant for song, found his charges couldn't grasp the bamboozling theories in his university courses. So he simply put the complex formulae and equations, to music. Now students flock to his classes — and they're getting better marks.

His students learn to memorise the value of Pi to the tune of Don McLean's *American Pie*, they're taught the features of a graph to Bette Midler's *From A Distance* and, for those interested in the mathematics of random occurrences such as lotteries it's *The Gambler* which they gleefully sing along to.

Dr Lesser said his two-minute maths ditties were generally the only thing a student would recall from a lesson — sometimes the entire semester's course.

"Although music does help

with memorisation, I use it more for motivation," said Lesser, a maths professor at Armstrong Atlantic State University in Savannah, Georgia.

"They're certainly outstanding mnemonic devices to help students learn and recall information, often with more enjoyment or less anxiety."

Dr Lesser chooses rap songs for memorising basic facts, but prefers longer, catchier tunes such as *We Will Graph You*, a reworking of Queen's classic *We Will Rock You*, (lead singer Freddie Mercury is pictured above) for learning fundamental theories such as the procedures needed for graphing a line or parabola.

But Dr Lesser won't whack his precious words into any old song. He chooses his songs lovingly and avoids profane songs and sacred tunes, preferring instead to use chart toppers that his generally teenage students will know.

## The method

Freddie Mercury's algo-rhythmic vocals have been "re-purposed" by Larry Lesser, who teaches students to graph a given function using the tune of Mercury's *We Will Rock You*.

The students must chant and clap the song in unison.

Lesser's bizarrely effective rendition is called *We Will Graph You* and goes like this:

*You wanna draw a parabola from the general form algebra.*

*Don't despair, complete the square. The x of the vertex comes out there, and we will, we will graph you! (chorus twice repeated).*

*The vertex now is figured out, but does the graph smile or frown?*

*The number next to the square of x, gave the sign to make the sketch, now — we will, we will graph you! (twice repeated).*

*Find and plot any x-intercepts, the constant c is the y-intercept.*

*A vertical line through the vertex, gives symmetry for your sketch, and we have, we have graphed you! (twice repeated).*

# “American Pi” prologue

lyric © 1997-2015 Lawrence M. Lesser

A long, long time ago, I can still remember  
 How that math sure used to make me smile.  
 And I knew if I had the chance,  
 I would ace geometry class  
 And make my parents happy for awhile.

But some math books made me shiver—  
 Dry procedures, all delivered:  
 Nothing past the rational,  
 and nothing transcendental.



I can't remember if I cried, reading 3.14159...

But something touched me deep inside the day I learned of pi.

2. Find the unknown side and the perimeter of each of the following rectangles.

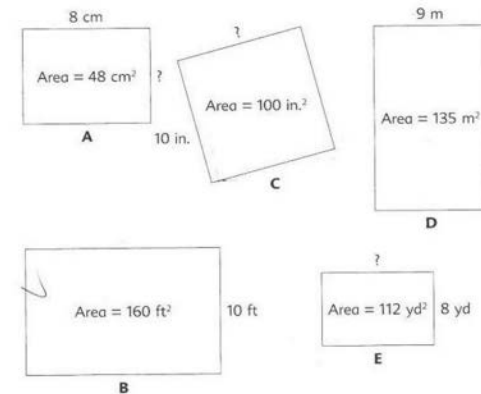


Figure	Area	Length	Width	Perimeter
A	48 cm <sup>2</sup>	8 cm	6	28 cm
B	160 ft <sup>2</sup>	16	10 ft	52 ft
C	100 in. <sup>2</sup>	10 in.	10 in.	40 in.
D	135 m <sup>2</sup>	15 m	9 m	48 m
E	112 yd <sup>2</sup>	14	8 yd	44

# “American $\pi$ ”

lyric © 1997-2015 Lawrence M. Lesser

## *CHORUS:*

Find, find the value of pi,  
starts 3 point 1 4 1 5 9...

A good ol' fraction  
you may hope to define  
but the decimal never dies,  
never repeats or dies....

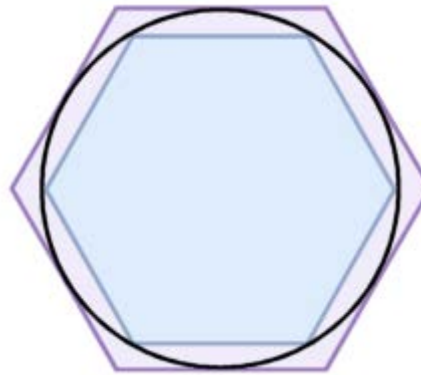
# “American Pi”

lyric © 1997-2015 Lawrence M. Lesser

**1 Kings 7:23**

In the Bible, we do see  
the circle ratio appears as 3,  
or a little more....

That genius Archimedes found  
with polygons, an upper bound  
Of  $22 / 7$  for sure!



$$\frac{4}{\pi} = 1 + \frac{1}{3 + \frac{4}{5 + \frac{9}{7 + \frac{16}{9 + \frac{25}{11 + \frac{36}{13 + \dots}}}}}}$$

The Chinese got it really keen: 355 over 113!

More joined the action with series and continued fractions.

In the 1700s, my oh my, the English coined the symbol  $\pi$ ,

Then Lambert proved that you and I can't find rational pi.

He started singing.....

# “American $\pi$ ”

lyric © 1997-2015 Lawrence M. Lesser

## *CHORUS:*

Find, find the value of pi,  
starts 3 point 1 4 1 5 9...

A good ol' fraction  
you may hope to define  
but the decimal never dies,  
never repeats or dies....



# “American Pi”

lyric © 1997-2015 Lawrence M. Lesser

Late 1800s, Lindemann shared why a circle can't be squared  
But some folks tried anyway—

Like the Indiana doctor who said pi was 4 or 3.2

And thought his proof should be a law someday.

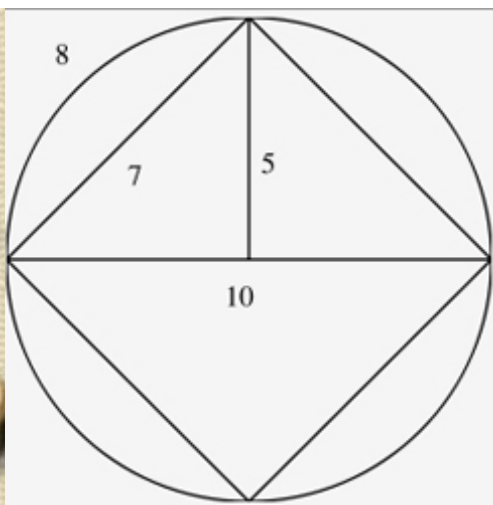
The Indiana congressmen read his paper there and then--

A bill got through the House by a vote unanimous!

But in the end, the statesmen cried, “It's not for us to decide,

So the bill was left to die like the quest for rational pi.

Let's try singing....



# “American $\pi$ ”

lyric © 1997-2015 Lawrence M. Lesser

## *CHORUS:*

Find, find the value of pi,  
starts 3 point 1 4 1 5 9...

A good ol' fraction  
you may hope to define  
but the decimal never dies,  
never repeats or dies....

my version of “Mean”,  
2012 Grammy-winning country song of the year

You...with your words like mean & mode &  
symbols that you use against me--  
 $\mu$ ....stands for a population mean  
and  $\bar{x}$  for the sample.

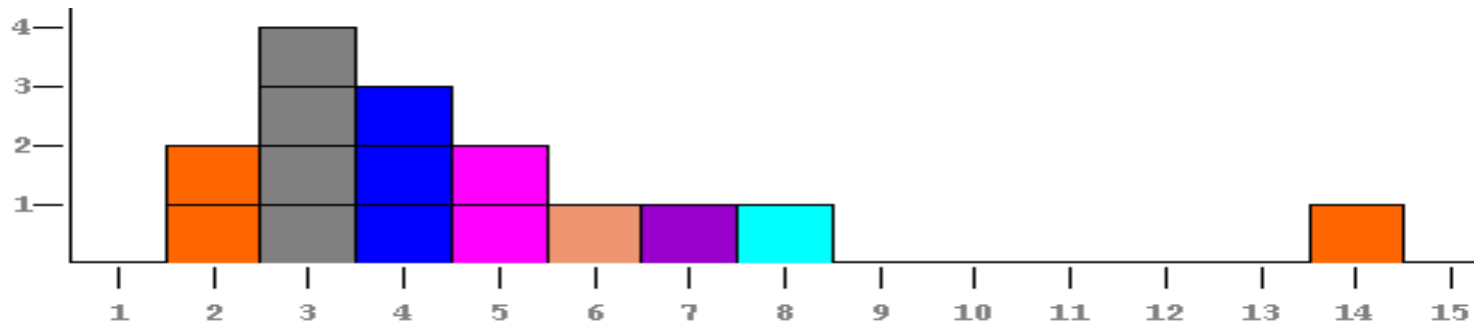
Who...knows what symbol to use  
for other measures of data location?  
You....pickin’ each time the mean.



# “Mean”

(lyric © 2011 L. Lesser)

But it can get pulled off  
by just one single point,  
And you don't know what you don't know...

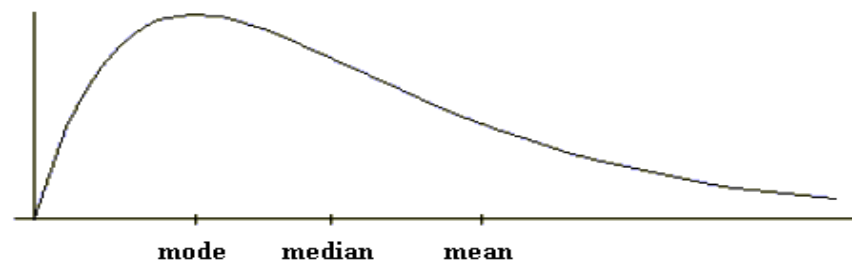


Show:

Mean

Median

Mode



“Mean” (lyric © 2011 L. Lesser)

Sometimes, data have a real asymmetry

Or values that are really quite extreme. Yeah yeah

In those cases, better use the median

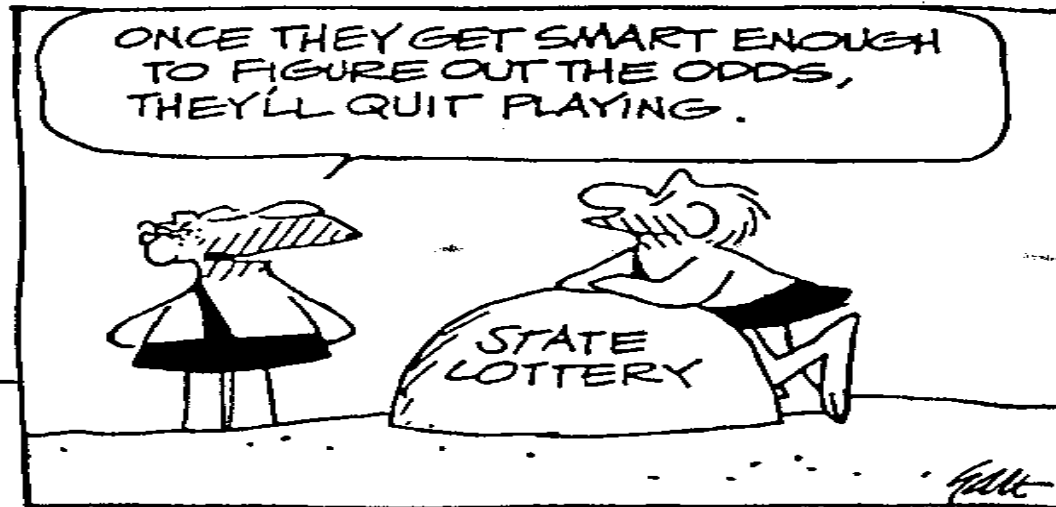
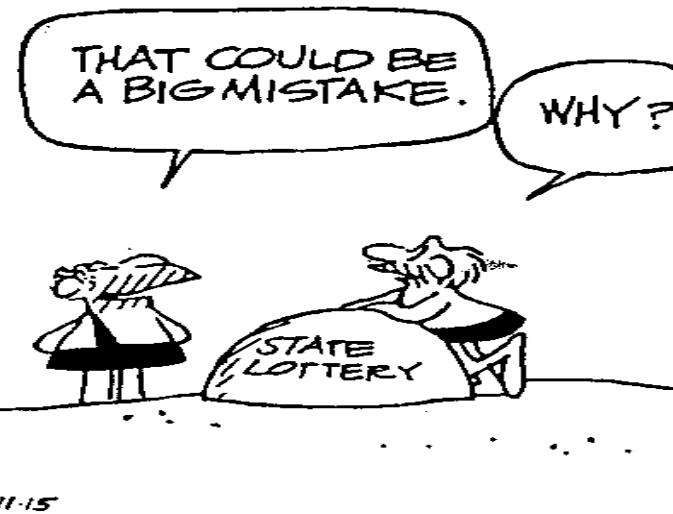
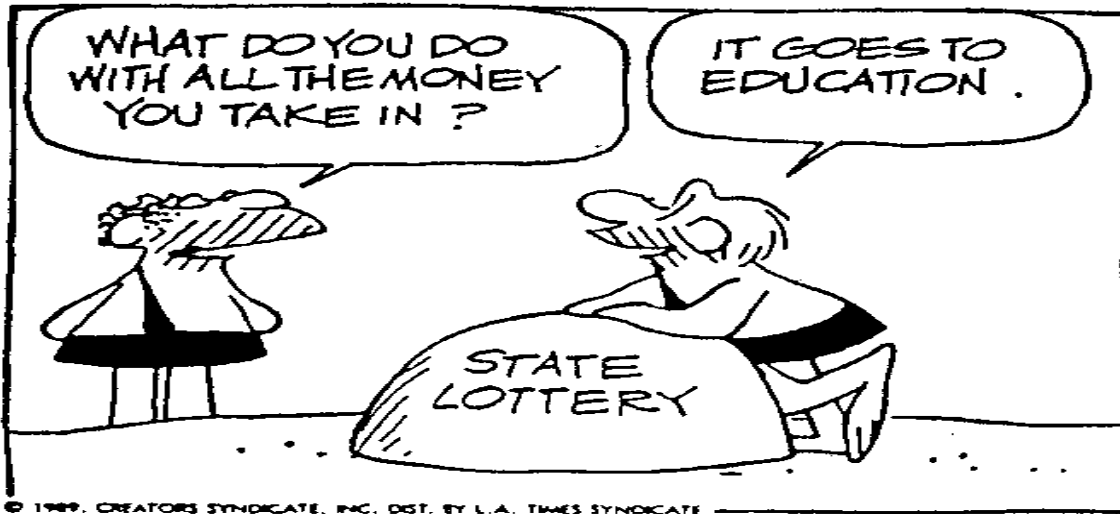
If what's typical is what you need –

That's not gonna be the mean!

That won't be the mean with outliers,

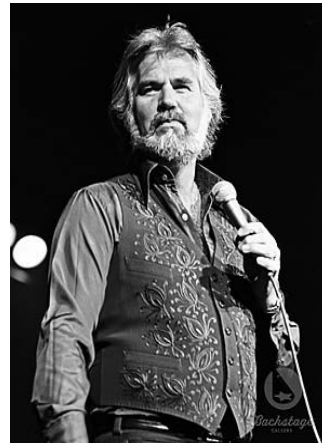
just forget it, do you see just what I mean

& mean & mean & mean....



# lotta lottery outreach

- adult ed. courses
- media coverage (newspapers, radio/TV from local to CNN)
- features in teacher-friendly journal (e.g., *Sept. 2012 Mathematics Teacher*)
- Lottery Literacy webpage; YouTube video
- a song: #1 country song whose title begged to be made into a lottery education song!



## Professor seeks to even odds of lottery

■ Class aims to unravel the numerical mysteries of the state's game of chance

By David Elliot  
American-Statesman Capitol Staff

What are the chances of winning the Texas Lottery jackpot if you play 20 times a week for 50 years? Are the numbers 1-2-3-4-5-6 as likely to come up as, say, 6-7-15-20-36-45? Why does the Texas Lottery give \$3 prizes to players who match three numbers?

These are some of the sample questions that a local statistics professor hopes to answer when he offers an informal, not-for-credit class entitled, "Lotto Luck," next month at the University of Texas. The class, open to 15 students on a first-come, first-served basis, will be 10 a.m. to noon on Sept. 18 on the UT campus.

Planned by St. Edward's University Professor Larry Lesser, the class is not intended to encourage or dissuade anyone from playing the lottery. Rather, Lesser merely wants people to know the odds they're up against — and perhaps more important, to be able to understand odds and probabilities.

"People" talk about illiteracy in



# “The Gambler” excerpt © 2001, 2009 L. Lesser

On a warm summer’s evenin’,  
on a train bound for nowhere,  
I met up with a gambler --  
we were both too tired to sleep.  
So he told me how he planned  
winnin’ lottery prizes



‘Til, as a math teacher, I just had to speak:

“Son, you track those draws,  
you say ya got a system—  
You call some numbers “hot”,  
you deem others “due”;  
But I insist, they each have  
the same chance—  
If you’re gonna play the game, boy,  
Ya gotta know what’s true!”

## Lotto Texas® Past Winning Numbers

[2014](#) [2013](#) [2012](#) [2011](#) [2010](#) [2009](#) [2008](#) [2007](#) [2006](#) [2005](#) [2004](#) [2003](#) [2002](#) [2001](#) [2000](#)  
[1999](#) [1998](#) [1997](#) [1996](#) [1995](#) [1994](#) [1993](#) [1992](#) [All Years](#) [Download](#) [View In Draw Order](#)  
[Print Friendly Format](#)

Draw Results	Draw Date	Winning Numbers	Estimated Jackpot	Jackpot Winners	Jackpot Option
<a href="#">View Details</a>	02/12/2014	13 16 31 36 43 49	\$9 Million	Roll	
<a href="#">View Details</a>	02/08/2014	7 16 17 26 42 51	\$8.25 Million	Roll	
<a href="#">View Details</a>	02/05/2014	16 23 25 29 49 54	\$7.5 Million	Roll	
<a href="#">View Details</a>	02/01/2014	6 10 14 27 36 51	\$7 Million	Roll	
<a href="#">View Details</a>	01/29/2014	8 19 43 48 49 51	\$6.75 Million	Roll	
<a href="#">View Details</a>	01/25/2014	4 25 27 36 44 48	\$6.5 Million	Roll	
<a href="#">View Details</a>	01/22/2014	4 10 13 25 34 41	\$6.25 Million	Roll	
<a href="#">View Details</a>	01/18/2014	16 18 32 36 45 46	\$6 Million	Roll	
<a href="#">View Details</a>	01/15/2014	23 25 28 35 36 47	\$5.75 Million	Roll	
<a href="#">View Details</a>	01/11/2014	12 20 23 30 40 41	\$5.5 Million	Roll	
<a href="#">View Details</a>	01/08/2014	8 19 25 26 30 39	\$5.25 Million	Roll	
<a href="#">View Details</a>	01/04/2014	14 21 27 35 37 38	\$5 Million	Roll	
<a href="#">View Details</a>	01/01/2014	15 17 43 45 47 50	\$12 Million	1	CVO



“The Gambler” © 2001, 2009 L. Lesser

*SING ALONG:* You gotta  
Know when you pick ‘em,  
What’s superstition,  
Know what is strategy  
And know when there’s *none!*

You never try to learn this  
At the 7-11:

Take the time right now for learnin’  
When the singin’s done!

*[now, watch my brief ‘steel guitar solo’....]*



# QUIZ

what happens if you  
give this hotel sign  
a quarter-turn?





**German mathematician,  
1862-1943**

David Hilbert

the “**Hilbert Hotel**”  
is no Hilton Hotel:

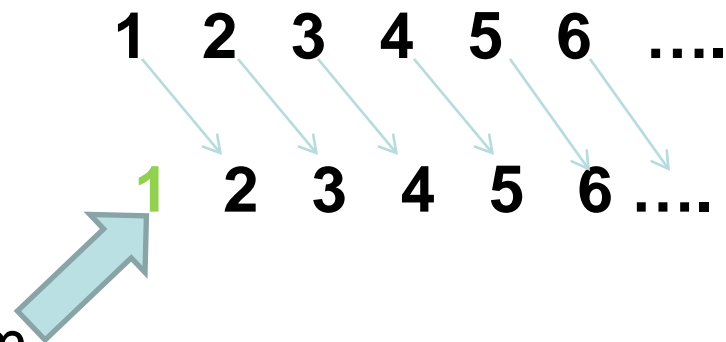
infinite # of rooms always full,  
but always room for more guests!



On a dark desert highway –  
not much scenery  
Except this long hotel  
stretchin' far as I could see.  
Neon sign in front read **NO VACANCY**  
But it was late and I was tired,  
so I went inside to plea.



The clerk said, “No problem.  
Here’s what can be done—  
We’ll move those in a room  
to the next higher one.  
That will free up the first room  
and that’s where you can stay.”  
I tried understanding *that*  
as I heard him say:



“Hotel (Called) Infinity” lyric © 2000-2012 L. Lesser

## ***CHORUS (sing along!)***

“Welcome to the **Hotel called Infinity**

Where every room is full

*(every room is full)*

Yet there’s room for more.

Plenty of room at the **HOTEL called Infinity**

Move ‘em down the floor

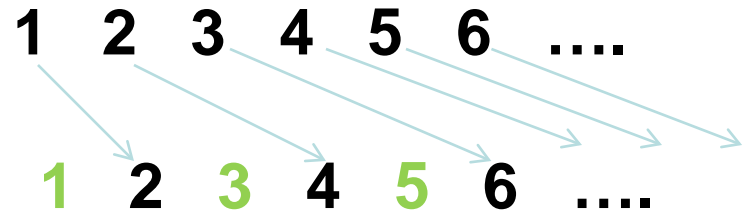
*(move em’ down the floor)*

To make room for more.”



# “Hotel (Called) Infinity” © 2000-2012 L. Lesser

My mind got more twisted  
when I saw a bus without end  
With an *infinite* number of riders  
coming up to check in!



“Relax,” said the nightman.  
“Each one will move  
to the double of their room number  
to free the odd-numbered rooms.”

Last thing I remember at the end of my stay—  
It was time to pay the bill, but I had no means to pay.  
The man in room #3 smiled, “Your bill is on me.  
#4 pays mine, and so on, so you get yours for free!



“Hotel (Called) Infinity” lyric © 2000-2012 L. Lesser

## ***CHORUS (sing along!)***

“Welcome to the **Hotel called Infinity**

Where every room is full

*(every room is full)*

Yet there’s room for more.

Plenty of room at the **HOTEL called Infinity**

Move ‘em down the floor

*(move em’ down the floor)*

To make room for more.”



# QUIZ

Speaking of infinity,  
what's the **world's longest math song**?

*Hint:* it uses “aleph null” ( $\aleph_0$ ),  
which is the cardinality of  $\{1, 2, 3, \dots\}$



“World’s Longest Math Song” (author unknown)

**$x_0$  bottles of beer on the wall**

**$x_0$  bottles of beer**

**Take one down,**

**Pass it around....**



**$x_0$  bottles of beer on the wall!**

***(repeat over & over & over & ....)***

*(see Donald Byrd’s variations in Sept. 2010 Math Horizons!)*

World's Longest Math Song: "all-ages version"

$x_0$  bottles of  $\sqrt{\text{beer}}$  on the wall

$x_0$  bottles of  $\sqrt{\text{beer}}$

Take one down,

Pass it around....

$x_0$  bottles of  $\sqrt{\text{beer}}$  on the wall

# in July 2016 *J. of Statistics Education*

JOURNAL OF STATISTICS EDUCATION  
2016, VOL. 24, NO. 2, 54–62  
<http://dx.doi.org/10.1080/10691898.2016.1190190>



OPEN ACCESS

## Assessing Fun Items' Effectiveness in Increasing Learning of College Introductory Statistics Students: Results of a Randomized Experiment

Lawrence M. Lesser<sup>a</sup>, Dennis K. Pearl<sup>b</sup>, and John J. Weber, III<sup>c</sup>

<sup>a</sup>Department of Mathematical Sciences, The University of Texas at El Paso, El Paso, Texas, USA; <sup>b</sup>Department of Statistics, The Pennsylvania State University, University Park, Pennsylvania, USA; <sup>c</sup>Department of Mathematics, Computer Sciences, and Engineering, Perimeter College at Georgia State University, Clarkston, Georgia, USA

### ABSTRACT

There has been a recent emergence of scholarship on the use of fun in the college statistics classroom, with at least 20 modalities identified. While there have been randomized experiments that suggest that fun can enhance student achievement or attitudes in statistics, these studies have generally been limited to one particular fun modality or have not been limited to the discipline of statistics. To address the efficacy of fun items in teaching statistics, a student-randomized experiment was designed to assess how specific items of fun may cause changes in statistical anxiety and learning statistics content. This experiment was conducted at two institutions of higher education with different and diverse student populations. Findings include a significant increase in correct responses to questions among students who were assigned online content with a song insert compared with those assigned content alone.

### KEY WORDS

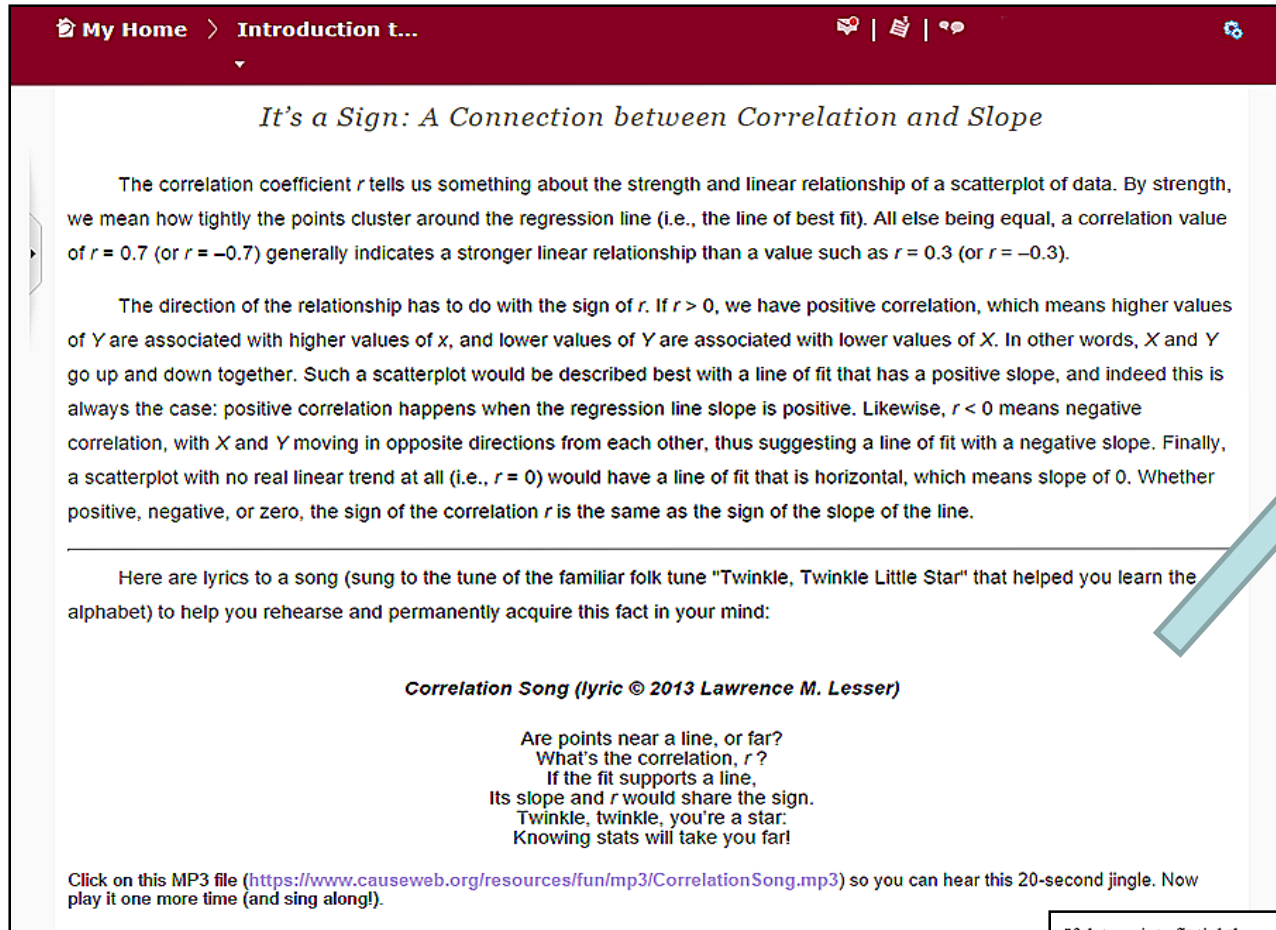
CAUSEweb fun collection;  
Humor; Song; Statistics  
education research

# diverse settings of Project UPLIFT's student-randomized experiment

	Two-Year College	University (medium-size)
Type of course	Statistical literacy	Statistical literacy
Region of U.S.	Southeast	Southwest
Student population	mostly Black	mostly Hispanic
Main audience	General education	Pre-service teachers
Text	Sullivan (2014) <i>Fundamentals of Statistics: Informed Decisions Using Data</i>	Utts (2005) <i>Seeing Through Statistics</i>
LMS (Learning management system)	Desire2Learn	Blackboard

# LMS screenshot (Desire2Learn) of a mini-reading with fun item insert

(half the students were randomized to receive the same reading without the insert)



**My Home > Introduction t...**

### *It's a Sign: A Connection between Correlation and Slope*

The correlation coefficient  $r$  tells us something about the strength and linear relationship of a scatterplot of data. By strength, we mean how tightly the points cluster around the regression line (i.e., the line of best fit). All else being equal, a correlation value of  $r = 0.7$  (or  $r = -0.7$ ) generally indicates a stronger linear relationship than a value such as  $r = 0.3$  (or  $r = -0.3$ ).

The direction of the relationship has to do with the sign of  $r$ . If  $r > 0$ , we have positive correlation, which means higher values of  $Y$  are associated with higher values of  $x$ , and lower values of  $Y$  are associated with lower values of  $X$ . In other words,  $X$  and  $Y$  go up and down together. Such a scatterplot would be described best with a line of fit that has a positive slope, and indeed this is always the case: positive correlation happens when the regression line slope is positive. Likewise,  $r < 0$  means negative correlation, with  $X$  and  $Y$  moving in opposite directions from each other, thus suggesting a line of fit with a negative slope. Finally, a scatterplot with no real linear trend at all (i.e.,  $r = 0$ ) would have a line of fit that is horizontal, which means slope of 0. Whether positive, negative, or zero, the sign of the correlation  $r$  is the same as the sign of the slope of the line.

---

Here are lyrics to a song (sung to the tune of the familiar folk tune "Twinkle, Twinkle Little Star" that helped you learn the alphabet) to help you rehearse and permanently acquire this fact in your mind:

***Correlation Song (lyric © 2013 Lawrence M. Lesser)***

Are points near a line, or far?  
What's the correlation,  $r$ ?  
If the fit supports a line,  
Its slope and  $r$  would share the sign.  
Twinkle, twinkle, you're a star:  
Knowing stats will take you far!

Click on this MP3 file (<https://www.causeweb.org/resources/fun/mp3/CorrelationSong.mp3>) so you can hear this 20-second jingle. Now play it one more time (and sing along!).

If data points fit tightly around the line  $Y = 2 - 3X$ , we can conclude there is \_\_\_\_\_ correlation.

- A. positive
- B. negative
- C. zero
- D. more information needed

# student-randomized experiment (fall 2013)

(53 from comm. college; 194 from medium-sized university)

- Half the students randomized to always have “fun inserts (song, cartoon, etc.)” in content readings accessed via LMS
- All students take exams with 12-14 embedded multiple-choice items related to the content readings, such as:

If data points fit tightly around the line  $Y = 2 - 3X$ , we can conclude there is \_\_\_\_\_ correlation.

- A. positive
- B. negative
- C. zero
- D. more information needed

- All students asked to take pre/post surveys: attitude & anxiety

# % Correct with and without Song Inserts

Topic	Without song	With Song	Difference
Margin of error: down with $n$ down by $\sqrt{n}$	57.3%	61.3%	4.0%
	9.1%	10.0%	0.9%
Standard score	62.5%	75.0%	12.5%
<b>Correlation &amp; slope</b>	<b>60.2%</b>	<b>73.8%</b>	<b>13.6%</b>
Equiprobability bias	40.9%	50.0%	9.1%
Multiplicity	36.1%	37.0%	0.9% (medium university)
$p$ -value	44.4%	50.0%	5.6% (2-yr. college)
<b>OVERALL</b>	42.3%	50.0%	<b>7.7%</b>
80% CI on total difference (2.8%, 12.6%) $p$ -value $\approx$ 0.04			

# findings of experiment

(see July 2016 *Journal of Statistics Education*)

- **Learning:** embedded questions on items related to **songs** were answered correctly 50.0% of the time by experimental group ( $n = 80$ ) and 42.3% by control group ( $n = 88$ ); 2-tailed  $p = .04$
- **Attitudes** (\*post-SATS): Little/no difference between experimental ( $n = 44$ ) and control ( $n = 44$ ) groups; not surprising since attitudes are more stable than anxiety
- **Anxiety** (\*post-SAM): no significant difference between experimental ( $n = 53$ ) and control ( $n = 59$ ) for the small sample sizes, but trending towards positive effect

\* = not enough students took pre AND post  
to test pre-post differences with adequate power



# part of our larger ongoing efforts...

<http://www.math.utep.edu/Faculty/lesser/Fun.html>

- Nov. 2008 *J. of Statistics Education*: review paper of 20 modalities of fun (e.g., humor, **songs**, games, cartoons) in statistics education
- March 2013 *J. of Statistics Education*: survey of 249 statistics instructors on **hesitations** and motivations for using fun
- **Spring 2014 *J. of Mathematics and the Arts*: rationale, strategies, and resources for using lyrics in mathematics/statistics class**
- June 2015 *Transformative Dialogues*: **case study** of an instructor's use of fun (including **song**) in statistics
- July 2016 *J. of Statistics Education*: randomized experiment on fun **inserts** (e.g., **songs** or cartoons) into intro statistics readings in LMS
- June 2016: *To Improve the Academy*: survey on use of **song** in **educational development**