Slices of Pi:
Rounding Up Ideas for Celebrating Pi Day

Every March 14 (3-14) is Pi Day! This is a great vehicle (if an "irrational" reason) for classes to have fun as they experience some interesting mathematics, especially mathematics connected to the number π. Teachers who assume this commemoration is obscure may be shocked to see all the references and resources on the Internet (e.g., using Google to search for "Pi Day" or from the list on p.2 of the January/February 2004 *NCTM News Bulletin*).

Celebrations are held not only in classrooms but also in other institutions coast-to-coast, ranging from San Francisco's Exploratorium to New York's Goudreau Museum of Mathematics in Art and Science. This article shares ideas to help teachers plan their own celebrations (for their clubs or classes or for their whole school) for Monday, March 14, 2005. While non-mathematical activities such as eating pie are universal, the content of the mathematical explorations is mostly for students at the secondary level except where noted.

Pi Day Entertainment

The most popular time for a Pi Day party is 1:59pm (3.14159...), a good time for some kind of afternoon snack. The obvious choice for refreshments is, of course, several kinds of pie (perhaps decorated with the letter pi or some digits of pi, or with the diameter and circumference outlined with some topping or icing), perhaps accompanied by a pie-making or pie-eating contest. Or a contest to carve the most mathematical pumpkin ("pumpkin pi"). One could assemble a veritable pi picnic of munchies that begin with the letters "pi": pizza pie, pineapple(be sure to note the Fibonacci numbers in the spirals before it's cut), pickles, pistachios, pinto beans, pimento, pita, pilaf.

The event can also include mathematical entertainment related to the number π, such as music (perhaps played on a pi-ano). One high school math club composed a song by assigning the first 50 digits in π to the key of C, letting 1 = middle C, 2 = the D above middle C, etc. (Lewellen 1987). Other melodies based on pi's digits can be found on the Internet. Also, Lesser (2000, 2003) offers an informative and playful lyric "American Pi" whose chorus contains a mnemonic for the first 6 significant figures:

> "Find, find the value of pi —-
> starts 3 point 1 4 1 5 9
> Good ol' boys gave it a try, but the decimal
> never dies, the decimal never dies..."

The lyric's three verses span historical highlights including a Biblical value and the Indiana legislature's 1897 consideration of a bill that declared π equal to 4 (Hallerberg 1977)! Students may sing the lyric to the tune of Don McLean's #1 1971 hit "American Pie" (though students may be more familiar with its return to the charts in 2000 thanks to Madonna). Teachers wanting to help students distinguish between the area and circumference formulas for a circle (apparently telling them to consider which one will have squared units isn't sufficient or most effective for some students) might offer this additional short song:

Circle Song (lyrics by Lawrence Mark Lesser; may be sung to the tune of "Twinkle, Twinkle Little Star")

> Take your finger round a jar
> Circumf'rence equals 2πr.
> For area, you multiply
> R squared by that number π.
> Twinkle, twinkle, you're a star
> Knowing math will take you far!

Bill Amend's nationally syndicated comic strip "FoxTrot" has actually had several (e.g., 11/25/00, 10/26/01, 1/13/04) strips whose punchlines are based on pi's having an infinite number of digits. Other jokes about π are related to specific circle formulas, such as these three (try making up your own!):

> "Do you know pi r squared, Grandpa?"
> "Nonsense! Pie are round, and cornbread are square!"

> "In the Greek alphabet, π is the sixteenth letter (and 16 is the square of 4). In the English alphabet,

p is also the sixteenth letter, and i is the ninth letter (the square of 3). Add them up (16+9), and you get 25 (the square of five). It's no wonder that they say: Pi are squared!"

"What do you get when you divide the moon's circumference by its diameter?"
"Pi in the sky!"
(alternative answer: "MoonPie")

Pi-related activities abound, in varying degrees of playfulness. The wall clock in a trigonometry classroom could be decorated to tell time in radians. A blindfolded cologne contest could be held to see if someone can recognize the woodsy fragrance called Pi (released in 1999 by Givenchy) or see if people rate it higher than other colognes. Also, students can take turns trying to break open a candy-filled π-shaped piñata. Or play ping-pong with homemade π-shaped paddles. Or do exercises with a Pi-lates ball. Or make the best pi sculpture out of sand or ice. Or make the best pi jewelry (I have seen π earrings and a π necklace) or painting a digit of pi on each fingernail. Maybe a session of "transcendental meditation" about the transcendental number pi. A poetry contest or reading could be held, including mathematical poems in Growney (2001) such as Wislawa Szymborska's "Pi". By the way, if a student happens to suggest showing the 1998 indie movie *Pi* (which won Darren Aronofsky the Director's Award at the Sundance Film Festival), teachers and math club sponsors should be advised *not* to show the movie (at least not without prior administrative and parent permission), as it is rated R for language and disturbing images.

Pi Day Contests

Perhaps a contest of who can recite the most digits of π (or come up with the best mnemonic for remembering them) or a trivia contest about pi (e.g., Eve Andersson's

http://eveander.com/trivia/

or facts from Beckman (1971), Schepler (1950), or Berggren, Borwein, & Borwein (2000), or Posamentier & Lehmann (2004)). Mnemonics are usually sentences where the number of letters in the nth word corresponds to the nth digit of pi (thought question: what happens at the 33rd digit?). Digit contests (or huge posters of the first several thousand digits) should be kept in perspective with the knowledge that knowing pi to as few as 50 digits more than suffices to estimate the circumference of a circle the size of the known universe to the accuracy of the size of a proton! As physics instructor Spenner (2003) explains: "The radius of a typical nuclear-bound proton is 10^{-15} meters. The universe has been expanding for about 15 billion years, so its edges are roughly 15 billion light years across, which works out to about 10^{26} meters, which is 10^{41} times the size of a proton." (Speaking of physics, March 14 is not only Pi Day, but also the birthday of Albert Einstein!)

Inspired by Waldner (1994), I launched a series of Pi Day events for Emery High School this year. One of the events featured "problems of the day" posted in the hallways for all to try. It is especially nice if such problems relate to circles or spheres, such as:

Which deal is better: an 8" $3.50 pizza or a 14" $10 pizza?

Imagine a wire snugly wrapped around the earth's surface at the equator. If the wire were made 100 feet longer to form a circle at a uniform height above the earth's surface all the way around, how high off the ground would the wire be?

Another event for the school was a contest in which each student could choose one of these three categories to enter:

- Essay: Double-spaced typed essay (<500 words) on the topic "Mathematics is Everywhere".
- Art: Art with a mathematical theme or constructed by mathematical principles (e.g., symmetry, perspective) or explicit formulas. This could include a variety of products such as "fractal art", tesselation art, etc. The final product might be two-dimensional (a poster or painting) or even a 3-D model/sculpture.
- Creative Writing: Lots of options (poem, lyric, etc.) as long as product has originality and mathematical theme or mathematical principle used in construction.

Some schools or math clubs have the tradition of scheduling local or regional interschool competitions sometime near Pi Day.

Pi Day Mathematical Explorations

Most teachers are familiar with an activity to estimate π empirically by having students measure (with a tape measure) the circumference and diameter of circular objects (e.g., jar lids) of various sizes, discovering that the ratio *C*/*D* is about the same for all of them (e.g., Barnard and Wheeler 2003). This could be done even by upper elementary or middle school students, while high school students could go on to plot the values of *C* versus *D* and use technology to find the line of best fit (ideally without intercept). The slope of this line of fit would then be an estimate of π (e.g., Pyzdrowski and Holtan 1996).

Students can recreate or explore some of the more historically famous activities involving π, such as Archimedes' classical method for estimating π. For example, consider a unit circle with inscribed and circumscribed hexagons. A geometry student can verify that those hexagons have perimeters of 6 and $4\sqrt{3}$, respectively. Bounding 2π between these numbers results in estimating pi as between 3.00 and 3.47. The approximation improves as the number of sides in the polygon is increased (e.g., repeatedly doubled).

A geometric probability or Monte Carlo method of estimating π can be implemented as follows. A unit circle can be inscribed in a square in which *x* and *y* each range from -1 to 1. Since the ratio of their areas is $\pi/4$, we can set $\pi/4$ equal to the proportion of randomly generated ordered pairs within the square that land within the circle. Solving this proportion will allow us to estimate π. On the TI-83, one way of generating each ordered pair (x,y) of values within the square is to use the sequence -1 + 2* MATH ⇒ PRB ⇒ rand ⇒ ENTER ⇒ ENTER. [Note: rand is a number randomly picked from the interval (0,1).] Every ordered pair (x, y) such that $x^2 + y^2$ is less than 1 falls inside the circle.

http://www.mste.uiuc.edu/activity/estpi/

Another probabilistic method is Buffon's 1777 experiment using needles dropped onto a floor ruled with parallel lines uniformly spaced A units apart (the probability that needles of length L land on top of one of these lines is $\frac{2L}{\pi A}$). (Note that we choose L < A so that a needle can't hit more than one line.) To illustrate, suppose 100 three-inch needles are dropped on lines 12" apart, so L = 3 and A = 12. Suppose 16 of those 100 needles land so that they intersect a line. Setting that empirical probability of .16 equal to the theoretical probability $\frac{2L}{\pi A}$ yields an estimate of 3.125 for π. In a secondary methods course I teach for preservice secondary teachers, we went out in the hallway, used only the horizontal lines of the square floor tiles, and obtained a reasonably accurate estimate of π by having several student groups repeatedly drop 10 popsicle sticks! The expression $\frac{2L}{\pi A}$ can be derived with first-year calculus and appears in several books (e.g., Eves 1990).

In addition to activities that estimate pi, students can also do explorations of famous formulas that involve pi. For example, students can dissect a circle into 16 congruent sectors, and arrange them in alternating side-by-side orientations (picture two interlaced rows of pointy teeth) to form an approximate parallelogram with area $\left(\frac{C}{2}\right)\left(\frac{d}{2}\right)=\left(\frac{2\pi r}{2}\right)r=\pi r^2$ (e.g., Barnard and Wheeler 2003). A video showing this in an animation sequence is *Story of Pi* by Project MATHEMATICS. You can order it or see excerpts at

http://www.projectmathematics.com/storypi.htm

Now we have a way to make sense out of the formula without having to bring out the tools of calculus to evaluate this integral:

$$2\int_{-r}^{r}\sqrt{r^2+x^2}\,dx$$

This video also has an animation sequence of Archimedes' method mentioned earlier.

Another activity allows students to predict and then measure how many "head-circumferences" (as measured by a tape measure horizontally around the head) tall they are. The typical result is 3, but students' predictions tend to be too high because they underestimate circumferences.

Ancient Connection: Biblical Pi

Students (pi-ous or not) are fascinated to learn that the value of pi is implied in the Bible (1 Kings 7:23): "He made the 'sea' [a copper tank for ritual immersion] of cast [metal] 10 cubits from its one lip to its [other] lip, circular all around, five cubits its height; a 30-cubit line could encircle it all around." After verifying that the passage most directly implies a value of 3, students can then discuss two interpretations, as have clergy and scholars (Tsaban and Garber 1998). Either 10 and 30 are approximations (students can discuss to what precision), or we must consider the tank's thickness (students can discuss how thick it would have to be) with 10 being measured outer lip to outer lip and the 30 measured around the inner surface. Also fascinating is a lesser-known gematria concerning the Hebrew word for "line" in the above passage. When that word is converted into the numerical equivalents of its written and spoken forms, respectively, the numbers 111 and 106 are obtained (Tsaban and Garber 1998). When 3 is multiplied by the ratio 111/106, the value obtained (3.14150943...) approximates π with accuracy greater than 99.997%!

Why the letter pi?

The first use of the Greek letter π to represent the ratio of a circle's circumference to its diameter seems to have been in the textbook *Synopsis Palmariorium Mathesios*, written by William Jones in 1706. He chose pi because it was the first letter of the Greek word 'perimetrog', meaning 'surrounding perimeter'.

- Usiskin, Z., Peressini, A. Marchisotto, E.A., and Stanley, D. (2003). Mathematics for high school teachers: An advanced perspective. Upper Saddle River, NJ: Prentice Hall.

Contemporary Literature Connection: *Life of Pi*

When I heard that Emery High School assigned the best-selling, award-winning book *Life of Pi* (Martel 2001) for all to read over the summer, I admit that the title made me think that it just might be a work (e.g., Blatner 1997) filled with mathematical beauty associated with the number pi or a book (e.g., Abbott 1976) that uses a mathematical framework as an allegorical vehicle to tell a sociopolitical story. While neither proved to be the case, this story of a zookeeper's son crossing the Pacific Ocean in a lifeboat with a Bengal tiger is enjoyable and interesting to read and did turn out to have occasional moments flavored by mathematics.

Having such a book assigned in your school's English classes may make for a more interdisciplinary Pi Day. Most of these questions can be answered just from the referenced excerpts so that this assignment does not exclude those who have not read the entire book, nor does it spoil the essence of the story for those about to read it. This list is designed to work either as a homework assignment or as a group activity that would take a full class period. Two Emery students, Brooke and Phillip, created impressive "Life of Pi posters" in which they brought these problems to life, artistically illustrated and adorned with a three-dimensional boat! Questions #1 and 6 specifically relate to pi or to circles, while the others address other aspects of mathematics.

Q1 Tired of being teased over his given name Piscine, the main character adopts Pi as his name. Discuss this passage from page 24: "In that Greek letter that looks like a shack with a corrugated tin roof, in that elusive, irrational number with which scientists try to understand the universe, I found refuge."

Q2 On pages 48-49, it says: "That which sustains the universe beyond thought and language and that which is at the core of us and struggles for expression, is the same thing. The finite within the infinite, the infinite within the finite." How might entities such as fractals relate to this?

Q3 Read pages 137-138 and make a reasonably accurate scale diagram of the lifeboat. Doing so is not only a good application of geometry, but also helps better visualize certain scenes and events of the story. (For something simply whimsical, try drawing Mr. Satish Kumar, who is described on page 25 as "geometric: he looked like two triangles, a small one and a larger one, balanced on two parallel lines.")

Q4 Read the information about the supply rations (starting on page 144) and create an original word problem involving them.

Q5 Discuss how Pi's "problem solving" on the boat compares to what "problem solving" in math.

Q6 On page 199, Pi asks: "If the horizon was two and a half miles away at an altitude of five feet, how far away was it when I was sitting against the mast of my raft, my eyes not even three feet above the water?" First, have students assess the accuracy of the statement in the first half of the question, assuming the earth to be a sphere of radius 3964 miles. Then, have students do a calculation to verify that standing up does not increase the visible horizon distance by as much as one might guess. [Hint: The only math required is the Pythagorean theorem, so the key step is to draw a diagram featuring a circle and a right triangle whose longer leg is the earth's radius, whose shorter leg (tangent segment) is the sight line and whose hypotenuse (secant segment) is the earth's radius plus the boy's height.]

Q7 Discuss this exchange from page 299:
"We find it very unlikely."
"So is winning the lottery, yet someone always wins."

When I discussed Q6 with Emery HS ninth grade geometry students, they were impressed with the practicality of being able to tell how far one can see when looking out to the horizon from a raft (or from a beach). We then opened up their (Holt, Rinehart, and Winston) textbook and found a problem with exactly the same idea applied to finding the effective signal range of a communications radio tower.

Coming Full Circle

There is richness and mystique surrounding the number pi -- the ratio that cannot be written as a fraction -- the number whose decimal has no end or pattern, yet is related to perfectly symmetrical circles and spheres. But Pi Day is about more than just the number pi and is for more than just so-called "math nerds" -- it is an affirmation that mathematics has not just utility, but also beauty, history, mystery, and joy.

Each teacher will have to select or adapt the ideas that work best in her classroom. For example, an elementary school teacher will not be able to use *Life of Pi*, but could instead use one of the books in the Sir Cumference series (e.g., Neuschwander 2002) or a book such as Ross (1992). Amidst some frivolity, we hope we have rounded up a variety of interesting and serious activities from which to choose. And may the heightened fun and inspiration of the day spill over into many mathematics lessons throughout the rest of the year. After all, math's not always a piece of cake, but sometimes it's as easy as pi.

■

Larry Lesser, Ph.D. • <Lesser@utep.edu>
Associate Professor • University of Texas at El Paso

References

Abbott, E. (1976). *Flatland: A romance of many dimensions*. Cutchogue, NY: Buccaneer Books.

Barnard, J. T., Wheeler, E. R. (2003). *Mathematics activities for teaching and learning* (11th ed.). Dubuque, Iowa: Kendall/Hunt Publishing.

Beckman, P. (1971). *A history of pi*. New York: St. Martin's Press.

Berggren, L., Borwein, J.; Borwein, P. (2000). *Pi: A source book* (2nd ed.). New York: Springer-Verlag.

Blatner, D. (1997). *The joy of pi*. New York: Walker and Company.

Eves, H. (1990). An *introduction to the history of mathematics* (6th ed.). Fort Worth, TX: Saunders College Publishing.

Growney, J. (June 2001). Numbers and faces: A collection of poems with mathematical imagery. *Humanistic Mathematics Network Journal* 24.

Hallerberg, A. (May 1977). Indiana's squared circle. *Mathematics Magazine*. *50* (3), 136-140.

Lesser, L. (May 2000). Sum of songs: Making Mathematics Less Monotone! *Mathematics Teacher*, *93*(5), 372-377.

Lesser, L. (March 2003). American pi. *Pi in the Sky*, p. 28. (also available via http://www.pims.math.ca/pi)

Lewellen, H. (April 1987). Sing pi [Reader Reflections letter]. *Mathematics Teacher*, *80* (4), 264.

Martel, Y. (2001). *Life of pi*. New York: Harcourt.

Neuschwander, C., Geehan, W. (2002). *Sir Cumference and the dragon of pi: A math adventure*. Watertown, MA: Charlesbridge Publishing.

Posamentier, A.S. & Lehmann, I. (2004). *Pi: A biography of the world's most mysterious number.* Amherst, NY: Prometheus Books.

Pyzdrowski, L., Holtan, B. (November 1996). Exploring pi using the computer in middle school mathematics. *School Science and Mathematics*, 378-81.

Ross, C.S. (1992). *Circles: Fun ideas for getting a-round in math*. Reading, MA: Addison Wesley.

Schepler, H.C. (1950). The chronology of pi. *Mathematics Magazine*. January–February: 165-70; March-April: 216-28; May-June: 279-83.

Spenner, C. (October 30, 2003). Personal communication [email] with the author.

Tsaban, B., Garber, D. (1998). On the rabbinical approximation of pi. *Historia Mathematica, 25*, 75-84.

Waldner, B.C. (February 1994). Pi Day. *Mathematics Teacher*, 87(2), 86-87.

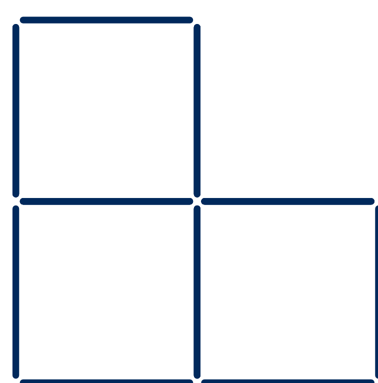
Puzzle Corner

Sticks #4 Puzzle

We are interested in how your students responded to this problem and how they explained or justified their reasoning. Please e-mail copies of your students' work, include your name, grade level, campus name and district name to Mary Alice Hatchett, Director of Publications *Texas Mathematics Teacher*. Selected submissions will be acknowledged and published in subsequent issues.

Please prepare a sketch of your solution

Arrange 10 craft sticks to form the following figure



Move two sticks to form four congruent rectangles.