Simulation:

To randomly pick 6 integers from 1 to 50 (without replacement) using TI-83,
MATH →PRB → randInt(1,50,6); ENTER or
APPS→ProbSim→RandomNumbers; Set(Numbers:6; Range:1-50; Repeat:No); Draw

Sample Space: collection of all possible outcomes for an experiment
(examples: if the experiment is drawing a card from a standard deck of cards, there are 52 possible outcomes; if the experiment is rolling one 6-sided die, there are 6 possible outcomes; if the experiment is rolling two 6-sided dice, there are 36 possible outcomes – imagine one red die and one green die and make a chart or table)

Fundamental counting rule (also known as the multiplication rule or basic principle of counting): Suppose that r actions are to be done in a definite order. Suppose there are $m_1$ possibilities for the first action, $m_2$ possibilities for the second action, etc. Then there are $m_1 m_2 m_3 \cdots m_r$ possibilities altogether for the sequence of r actions.

"k factorial" = k! = the product of the first k positive integers;
Example: 4! = (4)(3)(2)(1) = 24, which how many ways we can permute 4 objects.
By convention, we define 0! = 1

A permutation of r objects from a collection of m objects is any ordered arrangement of r of the m objects. (note: you can't pick the same object twice.) The number of these is denoted ${}_mP_r$ and equals m!/(m-r)!
Example: how many ordered arrangements of 2 objects can we make from a set of 4 objects?
EXCEL command =PERMUT(4,2)
TI-83 command: 4, MATH, PRB, nPr, 2, ENTER
(answer: 12; here's a list: ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc)

A combination of r objects from a collection of m objects is any (unordered) set of r of the m objects. (note: you can't pick the same object twice.) The number of these is denoted ${}_mC_r$ and equals m!/[r!(m-r)!]
Example: how many combinations of 2 objects can we make from a set of 4 objects?
EXCEL command =COMBIN(4,2)
TI-83 command: 4, MATH, PRB, nCr, 2, ENTER
(answer: 6; here's a list: ab, ac, ad, bc, bd, cd)

## Geometric random variable:

Table 14.2 of Section 14.5 shows a pattern that can be stated generally as follows: A geometric random variable X is the number of trials (each of which is independent and has probability p of success) for the first success to occur. The probability that X equals a specific number, such as k, is therefore given by the formula: $(1-p)^{k-1}p$. The values k can take on are 1,2,3,….. The expected value of X is 1/p. The sequence of probabilities P(X = 1), P(X = 2), P(X = 3), … is a geometric sequence (hence the name!). This is the only "memoryless" discrete random variable, so: P($X$ > 16 given that we already know that $X$ > 12) = P($X$ > 4).

Calculate the probability that the first success occurs on exactly the 8th trial when each trial has success probability of p = .40.
TI-83: 2nd, DISTR, geometpdf(.40, 8) ; EXCEL: =NEGBINOMDIST(7, 1, .40)

Example of "cumulative" probability (that X is less than or equal to k):
Calculate the probability that the first success occurs on or before the 8th trial when each trial has success probability of p = .40. TI-83 command: 2nd, DISTR, geometcdf(.40, 8)

# * Binomial random variable:

A binomial random variable X is the number of successes when there is a fixed number (n) of independent trials, each of which has a success probability of p (and therefore a failure probability of 1-p, based on Utts rule #1). It's called "BI-nomial" because each trial has only 2 possibilities (e.g., success or failure). The expected value of X is np. (make sense?). Since trials are independent, this is like sampling at random with replacement from a population.

Notice in the geometric random variable setup, there was only one success (and it was in the "last" position in the sequence of trials) and the number of trials was not fixed in advance. But now, the number of trials is fixed, and the number (and position) of successes are not fixed. Okay, so is the formula for exactly k successes (which means n-k failures) in n trials simply: $(1-p)^{n-k}(p)^k$ ? Not quite, because there are multiple sequences that result in a certain number of successes. For example, suppose we want the probability of exactly 3 successes in 5 trials if each trial has probability p. Since the order we get those successes does not matter, we would ask for the number of COMBINATIONS of 3 objects taken from 5 objects, which = 10 from what we learned before, so the total probability would add up to be $10(1-p)^{5-3}(p)^3$ . (Recall Utts' Rule 2, the addition rule for mutually exclusive events.) So the general ("pdf") formula for the probability of exactly k successes in n trials is $(n!/[k!(n-k)!])(1-p)^{n-k}(p)^k$.

Calculate the probability of exactly 2 successes in 10 trials when each trial has success probability of p = .30.
TI-83: 2nd, DISTR, binompdf (10, .30, 2) ; EXCEL: =BINOMDIST(2, 10, .30, false)

Examples of cumulative ("cdf") probability (that X is less than or equal to k):
Calculate the probability of 3 or fewer successes in 10 trials when each trial has success probability of p = .20.
TI-83: 2nd, DISTR, binomcdf(10, .20, 3) ; EXCEL: =BINOMDIST(3, 10, .20, true)

n = 10

| p / r | .01 | .05 | .10 | .20 | .30 | .40 | .50 |
|---|---|---|---|---|---|---|---|
| 0 | 0.9044 | 0.5987 | 0.3487 | 0.1074 | 0.0282 | 0.0060 | 0.0010 |
| 1 | 0.9957 | 0.9139 | 0.7361 | 0.3758 | 0.1493 | 0.0464 | 0.0107 |
| 2 | 0.9999 | 0.9885 | 0.9298 | 0.6778 | 0.3828 | 0.1673 | 0.0547 |
| 3 | 1.0000 | 0.9990 | 0.9872 | 0.8791 | 0.6496 | 0.3823 | 0.1719 |
| 4 | 1.0000 | 0.9999 | 0.9984 | 0.9672 | 0.8497 | 0.6331 | 0.3770 |
| 5 | 1.0000 | 1.0000 | 0.9999 | 0.9936 | 0.9526 | 0.8338 | 0.6230 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 0.9991 | 0.9894 | 0.9452 | 0.8281 |
| 7 | | | | 0.9999 | 0.9999 | 0.9877 | 0.9453 |
| 8 | | | | 1.0000 | 1.0000 | 0.9983 | 0.9893 |
| 9 | | | | | | 0.9999 | 0.9990 |
| 10 | | | | | | 1.0000 | 1.0000 |

Table from a reference or textbook appendix: