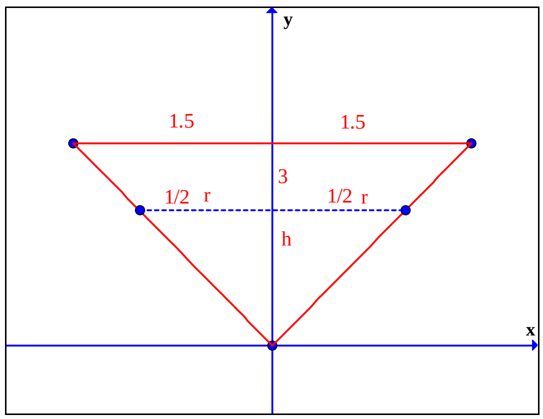
Problem Chapter 2.6 – 26.

A trough is 12 feet long and 3 feet across the top. Its ends are isosceles triangles with altitudes of 3 feet.

a) If water is being pumped into the trough at 2 cubic feet per minutes, how fast is the water level rising when the height is 1 foot deep?

Answer and Solution:

The cross section of the trough looks like the following:



Recall that in an isosceles triangle the altitude between the two equal sides bisects the opposite side. The statement "If water is being pumped into the trough at 2 cubic feet per minutes," can written as $\frac{dV}{dt} = 2$. The volume of the water is area of the cross section of water times the length of the trough (12 feet).

The volume of a triangle is $V = A(12) = \left[\frac{1}{2}(r)h\right](12)$. We need to rewrite the equation so that the only variable on the right side of the equation is h.

Using similar triangles, we have the following ratios, $\frac{r}{3} = \frac{h}{3}$. Therefore the relationship between r and h is $r = h$.

The equation $V = \frac{1}{2}(h)h(12) = 6h^2$. $\frac{d}{dt}[V] = \frac{d}{dt}\left[6h^2\right]$.

$$\frac{d}{dt}[V] = 12h\frac{dh}{dt}$$

$$2 = 12(1)\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{6} = 2 \text{ inches per minute}$$

b) If the same water is rising at a rate of $\frac{3}{8}$ inch per minute when the height (h) is 2 feet, determine the rate at which water is being pumped into the trough.

Answer and Solution:

Can't mix units, so convert $\frac{3}{8}$ inches to feet by multiplying by 1. $\frac{3 \text{ in}}{8} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{1}{32} \text{ ft}$

$$\frac{d}{dt}[V] = 12(2)\left(\frac{1}{32}\right)$$

$$\frac{dV}{dt} = \frac{3}{4} \text{ cubic feet per minute}$$