A new Analysis of Intermittence, Scale Invariance and Characteristic Scales applied to the Behavior of Financial Indices near a Crash

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This work is devoted to the study of the relation between intermittence and scale invariance, and applications to the behavior of Financial Indices near a crash. We developed a numerical analysis that predicts the *critical date* of a financial index, and we apply the model to the analysis of several Financial Indices. We were able to obtain optimum values for the *critical date*, corresponding to the most probable date of the crash. We only used data from before the true crash date in order to obtain the predicted critical date. The good numerical results validate the model.

Keywords: Econophysics, Scale Invariance, Intermittence, Stock Market Prices, Financial Indices, Crashes.

1-Introduction

During the last years the study of log-periodic structures and characteristic scales and the relation with the concept of scale invariance had grown due to the great amount of physical systems presenting log-periodic structures: fluid turbulence [1,2], diamond Ising model [3], earthquakes [4], materials rupture [5], black holes [6] and gravitational collapses [7] among others. In a mathematical context, we recall constructions as the Cantor Fractal [3][8], with a discrete scale changes invariant.

The presence of logarithmic periods in physical systems was noted by Novikov in 1966 [9], with the discovery of intermittence effect in turbulent fluids. The relation between both effects has been deeply studied, but it has not been formalized yet.

At the same time, the complexity of international finance has grown enormously with the development of new markets and instruments for transferring risks. This growth in complexity has been accompanied by an expanded role for mathematical models to value derivative securities, and to measure their risks. A new discipline, Econophysics, has been developed [10]. This discipline was introduced in 1995, see Stanley et all, [11]. It studies the application of mathematical tools that are usually applied to physical models, to the study of financial models. Simultaneously, there has been a growing literature in financial economics analyzing the behavior of major Stock Indices [10], [12-15].

A particular type of scale invariance is the one arising in the existence of intermittences or "stationary intervals", constant in the logarithm of the independent variable.

The functions that can be obtained from this analysis are:

$$f^{F}(x)=\beta e^{\alpha F(\log_a x)} \tag{3}$$

$$f^{C}(x)=\beta e^{\alpha C(\log_a x)} \tag{4}$$

Where β and α are real numbers, α is positive and $F(x)=I(x)$ and $C(x)=I(x)+1$ are the Floor and Ceiling functions respectively. Hence, the value obtained when applying the Floor function to a variable x will be the nearest entire number to x from the left, and the value obtained by the Ceiling function will be the nearest entire number to x from the right.

These two functions are discrete scale invariants, and more specifically, they satisfy equation (1) only when:

$$\lambda = a^{n}, \quad n \in Z \tag{5}$$

We recall that the conditions for a function to have discrete scale invariance, after we know that the system has intermittences are the following:

(i) The intermittence intervals must be constant in logarithmic scale, i.e. the steps have to be discrete:

$$d(\ln x)=K \tag{6}$$

where K is a positive number. Let $a=e^{K}$, then we have

$$d(\log_a x)=1 \tag{7}$$

Then, the interval of time of the intermittence is such that the logarithm of the variable x, in a basis a, has advanced one unit (in that period of time). Hence, we can conclude that due to the longitude of the intermittences there exists a basis in which the logarithm of the variable is equal to one.

(ii) The stationary intervals are consecutive, when one finishes, begins the next one.

(iii) The function such that its variable is discretized with the rule $d(\log_a x)=1$ is a power law, and the beginning and the end of a stationary interval have both to be a point of the function. We will call this function the basis function, which can be illustrated as follows:

As a financial index can be considered an asset, its (deterministic) financial behavior will be given by (9). Our hypothesis is that near a crash equation (9) is modified:

$$\frac{dS}{dT} = \mu \frac{S_c - S}{T_c - T} \qquad (10)$$

where T_c and S_c are, respectively, the time and the price for which the crash takes place. The heuristic analysis is as follows: near a crash there is a factor that produces a considerable increase in the index price, by the other hand, when the price is very near to the crash price, it has to exist another factor smoothing those variations, otherwise the crash would take place before the real date (for further details see [20]). Changing the variables in (10) we obtain that

$$\frac{dP}{dt} = \mu \frac{P}{t} \qquad (11)$$

where the variables are not anymore absolute data of the system: t and P are the distance to the critical time and critical price, respectively.

The second assumption will be that the temporal steps are discrete; therefore, the index evolution is not continuous and we have the intermittence phenomena. The index evolution is given by:

$$dt = Kt \qquad (12)$$

Then the frequency in the index price changes is proportional to the distance to the date in which the crash takes place. Equation (12) implies that

$$d \ln t = K \qquad (13)$$

From equations (11) and (13) we arrive to functions like (3) or (4). In this case we will work with function (3), due to the fact that the intermittences must take into account that the time approaches to the critical time from the right, because of the change of variables (11).

4- The data analysis methods: Analysis of the parameters and estimation of the critical time

We are plotting the t - p figure where t is the time distance from the crashing time and p is the price distance from the crashing price.

We can model the market behavior by using the following equation:

$$p_0 - f(t) = \beta e^{\alpha F(\log_a (t_0 - t))} \qquad (14)$$

We can see that $G(\cdot)$ is periodical with a period of $\ln a$.

To find α, we need to find the period of $G(\cdot)$. An ideal periodical function will have a spike in frequency domain at its period. So we use Fourier Transform and look for the frequency with a high power in the frequency domain.

A general Discrete Fourier Transform is given by

$$F(k) = \sum_{n=0}^{N} f(n) e^{-j2\pi nk/N} \tag{20}$$

In our method, we use the following transform

$$Q(k) = \sum_{n=1}^{N/2} G(x_n) e^{-j2\pi(k-1)(n-1)/(NK)} \tag{21}$$

where K is some constant that will determine the resolution of the Fourier Transform, and x_n, $n = 1,2,\cdots,N$ are the equal distant points between $\ln(t_0 - T_0)$ and $\ln(t_0 - T_1)$, where T_0 and T_1 are respectively the start and ending points of the observation window.

Notice that the original market data is daily price. When we use them in log-domain, the sample points of $G(\cdot)$ are no longer equal distant. However, (21) requires to evaluate $G(\cdot)$ at equal distant points x_n. To get the Fourier Transform, we use linear interpolation to evaluate $G(x_n)$. If we have $t_{m-1} < x_n \le t_m$, the interpolation is given by

$$G_{est}(x_n) = \frac{G(y_m) - G(y_{m-1})}{y_m - y_{m-1}} (x_n - y_{m-1}) \quad \text{for} \quad y_{m-1} < x_n \le y_m \tag{22}$$

where $y_m = \ln(t_0 - t_m)$.

We remark that the characteristic that the market must exhibit for assuming this approximation is that the behavior of the market is 'smooth' up to the crashing point. In other words, we assume the index curve is differentiable at each time point except for the crashing point. Therefore, we can use interpolation.

While higher order interpolation could be used, we found that a simple linear interpolation between two points is accurate enough for our purpose.

The Fourier Transform actually shows us the power at each frequency. Ideally, the power of the dominant frequency will be a local maximum. So we pick the frequency with a local maximum power within a roughly preset range. Once we have the frequency f, we can easily calculate the characteristic scalar a by

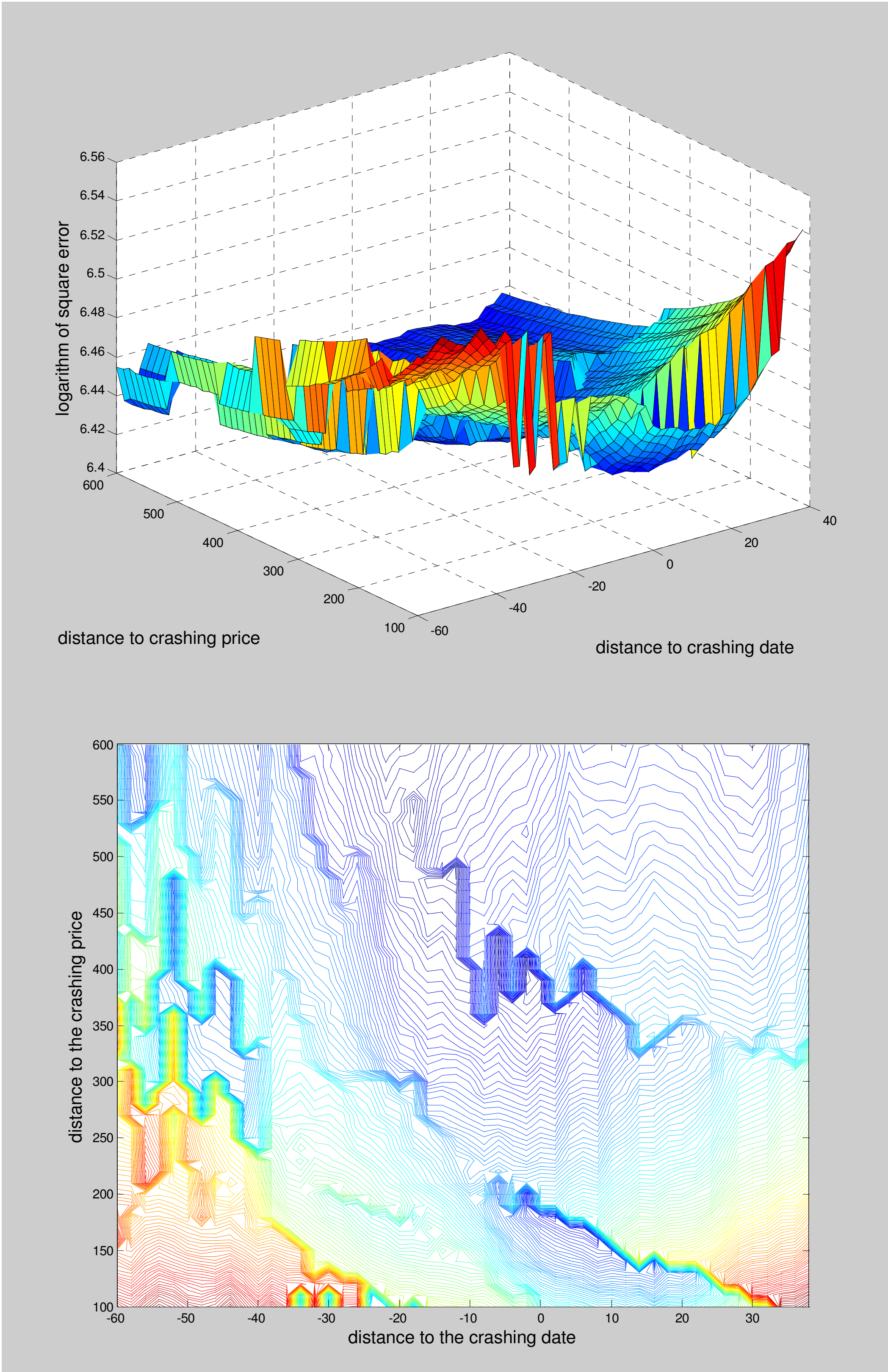

Figure 2: Error function of SP500 in log-domain

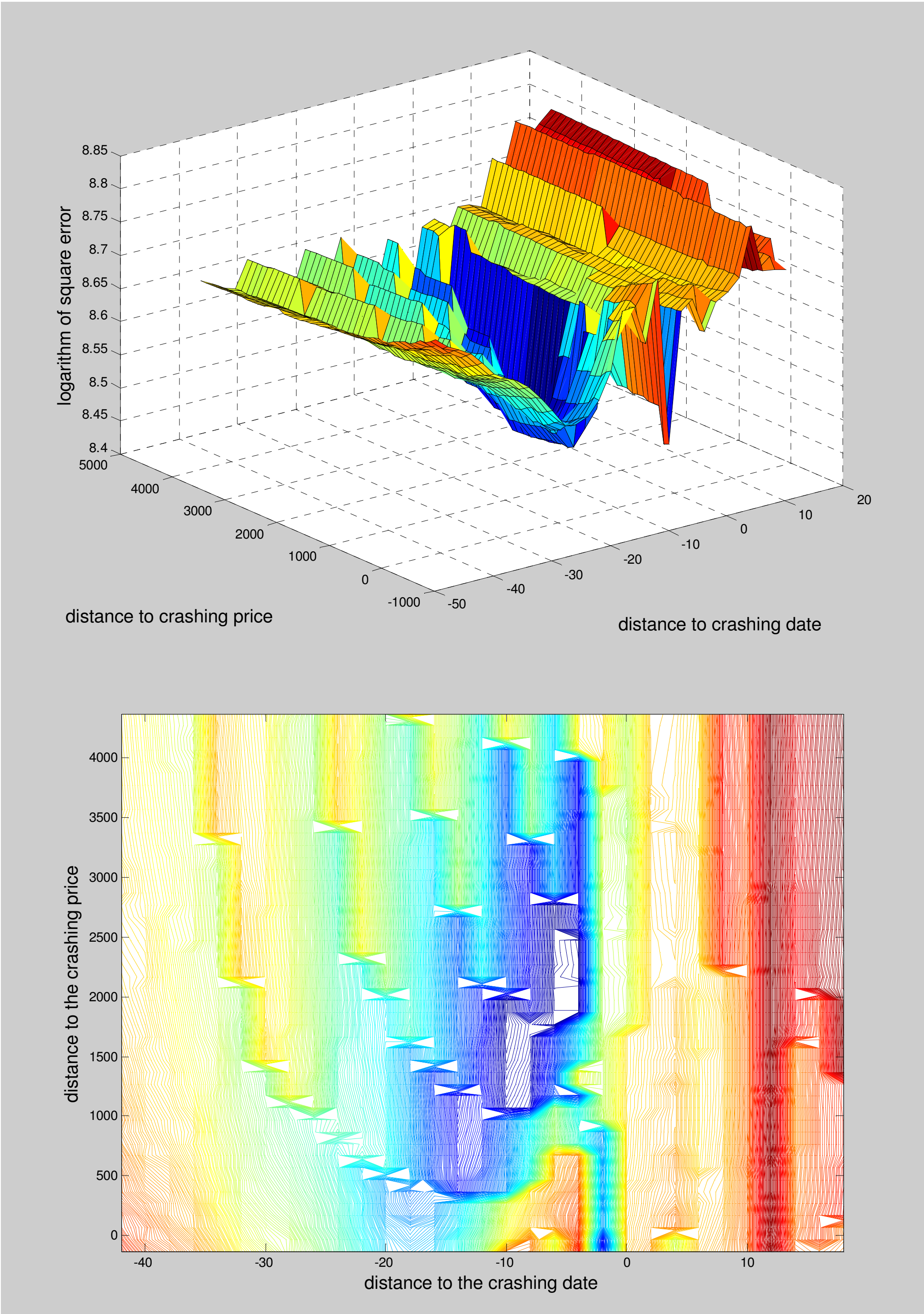

Figure 4: Error function of MXX in log-domain

6- Conclusions

The effects of certain local crisis on various and distant markets have largely been cited. The collapse of the crashes of 1987 (S&P500) dragged the collapse of markets worldwide. However not every crisis has sufficient strength as to drag the fall of leading indices in other countries. In [16] it has been shown that the crashes of Asian indices had consequences on emergent markets: the Asian crisis had sufficient strength as to drag the fall of leading Latin American indices.

Clearly all these indices crashed in similar dates due to a dragging correlated effect, which most likely started with the instability of the HSI index. This signals the likelihood of the events in different markets and different economic realities which strengthens the hypothesis of imitation and long range correlations among traders.

About the stability of the method, we want to remark that, even if the maximal and minimal values for the error are apparently similar, the graphs are plotted in log domain. Furthermore, we want to remark that we believe the noise in the market can actually change the crashing date, and the change may be significant.

We believe the market is a very complex system. The crash might be delayed due to some market force, so it happened sometime after the market top, but that's out the scope of this work. Our goal in this work is to develop a method for finding the market top which leads to a crash.

We also want to remark that we only use the data up to a couple of weeks before the crash in all the cases to estimate the critical time. The estimations have errors of around only 10 trading days. The excellent results validate the method.

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[19] L. Amaral, Comp. Phys. Comm. 121 1999, (145-152)

[20] Analysis of Intermittence, Scale Invariance and Characteristic Scales in the Behavior of Major Indices near a Crash.M. Ferraro, N. Furman, Y. Liu, M.C. Mariani, D.Rial. To appear in Physica A.

[21] "Options, Futures and Other Derivatives", John C. Hull –Prentice Hall Inc. 1997.

[22] "Dynamic Pricing Asset", Darell Duffie - Princeton University Press 1996.

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